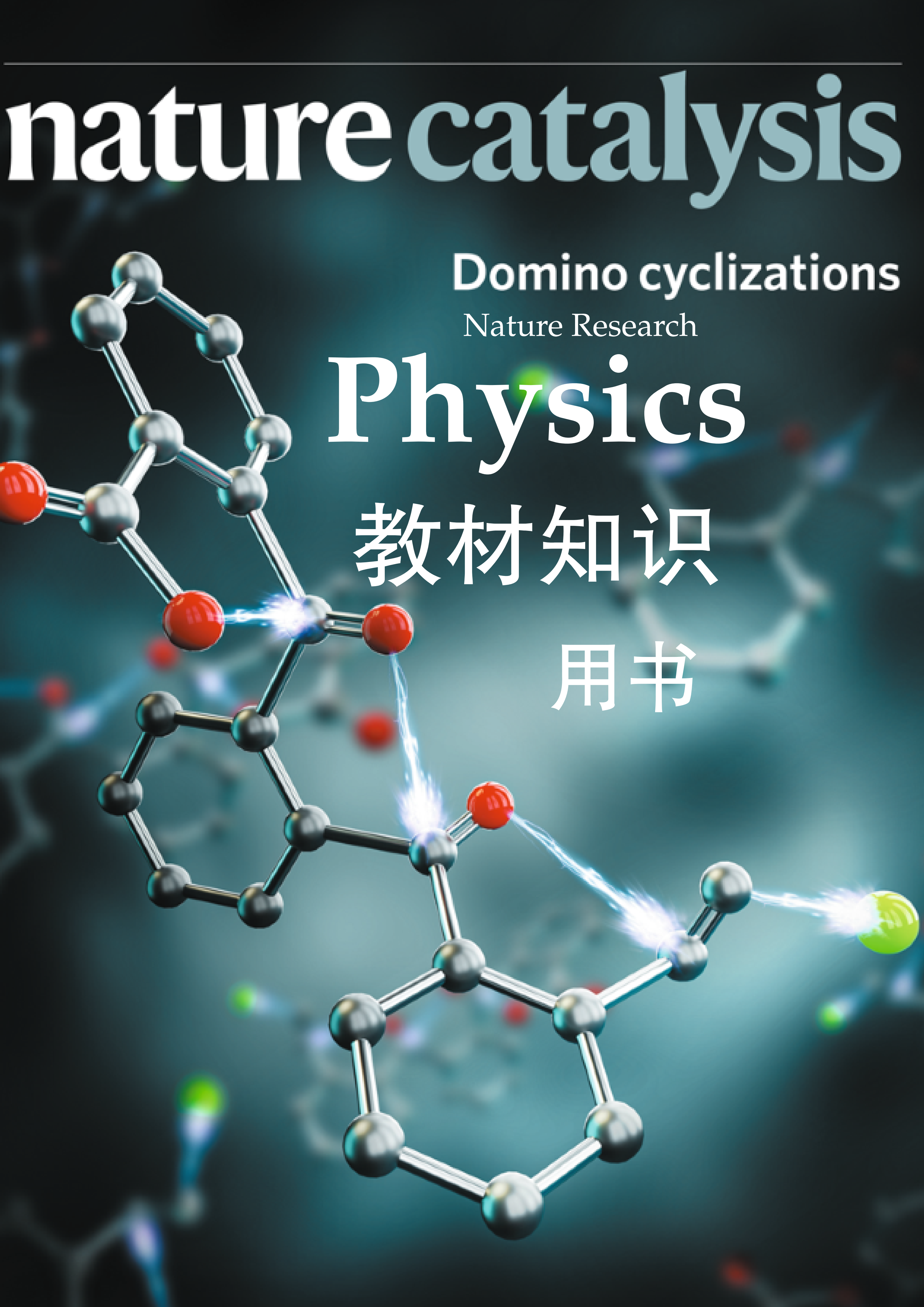
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内容简介

Last Update: 2021 年 2 月 9 日

Contacts

Original Author:
Email: colin-young@live.com
The latest update can be found via: https://github.com/yuhao-yang-cy/a2physics

Update version:

Email: http://www.latexstudio.net.
Picture of first page comes from https://mp.weixin.qq.com/s/1pzt7iykiYmq33-NMkE95g.
The latest update can be found via: https://github.com/LiuYongxue-code the course, I also recommend you to catch me right after any of my lectures.

关于本书模板

本书原来模板源于原作者 **colin-young** colin-young@live.com，因计划用于教师教材用书，故在其中设置了诸多的 tcolorbox 环境，并整理于：https://marukunalufd0123.hatenablog.com/entry/2019/03/15/071717.

Lecture Notes

These notes are supposed to be self-contained. I believe I have done my best to make the lecture notes reflect the spirit of the syllabus set by the Cambridge International Examination Board. Apart from the essential derivations and explanations, I also included a handful of worked examples and problem sets, so that you might get some rough idea about the styles of questions that you might encounter in the exams.

Throughout the notes, key concepts are marked red, key definitions and important formulas are boxed. But to be honest, the main reason that I wrote up these notes was not to serve any of my students, but just to give myself a goal. Since physics is such a rich and interesting subject, I cannot help sharing a small part of topics beyond the syllabus that I personally find interesting.

Also very importantly, I am certain that there are tons of typos in the notes. If you spot any errors, please let me know.

Literature

I borrow heavily from the following resources:

- Cambridge International AS and A Level Physics Coursebook, by *David Sang, Graham Jones, Richard Woodside* and *Gurinder Chadha*, Cambridge University Press
- International A Level Physics Revision Guide, by *Richard Woodside*, Hodder Education
- Longman Advanced Level Physics, by *Kwok Wai Loo*, Pearson Education South Asia
- Past Papers of Cambridge Internation A-Level Physics Examinations
- HyperPhysics Website: http://hyperphysics.phy-astr.gsu.edu/hbase/index.html
- Wikipedia Website: https://en.wikipedia.org

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目　录

第 1 章 Chapter 1
Circular Motion

1.1 Angular quantities

Angular quantities

movement or rotation of an object along a circular path is called **circular motion**

to describe a circular motion, we can use *angular quantities*, which turn out to be more useful than linear displacement , linear velocity , etc.

1.1.1 angular displacement

angular displacement

angular displacement is angle swiped out by object moving along circular

知识点解读

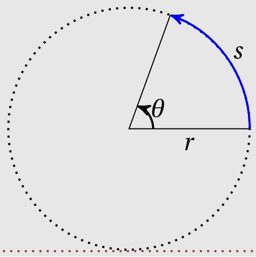
- unit: $[\theta] =$ rad (natural unit of measurement for angles)

conversion rule: 2π rad $= 360^\circ$

- if two radii form an angle of θ, then length of arc: $s = r\theta$

two radii subtending an arc of same length as radius form an angle of one **radian**

angular displacement is angle swiped out by object moving along circular



1.1.2 angular velocity

知识概念

angular velocity describes how fast an object moves along a circular path

重要概念

angular velocity is defined as angular displacement swiped out per unit time: $\boxed{\omega = \dfrac{\Delta\theta}{\Delta t}}$

- unit of: $[\omega] =$ rad s^{-1}, also in radian measures
- angular velocity is a *vector* quantity

this vector points in a direction normal to the plane of circular motion

but in A-level course, we treat angular velocity as if it is a scalar

angular velocity and angular speed may be considered to be the same idea

定理与公式推导

n interval Δt, distance moved along arc

$$\Delta s = v\Delta t = r\Delta\theta \quad \Rightarrow \quad \omega = \frac{\Delta\theta}{\Delta t} = \frac{v}{r} \quad \Rightarrow \quad \boxed{v = \omega r}$$

this relation between linear speed and angular speed holds at any instant

The vector points in the direction perpendicular to the circular motion plane, but in the A-level course, we treat the angular velocity as a scalar , That is, when we consider angular velocity, we regard it and linear velocity as the same physical quantity to describe the most circular motion of an object.

For the constant relationship between linear velocity and angular velocity, we can use linear velocity to describe the angular velocity, and conversely, we can use angular velocity to describe linear velocity.

1.1.3 Uniform circular motion

➤ 定义与概念

when studying linear motion, we started from motion with constant velocity v

consider the simplest possible circular motion $\longrightarrow$ circular motion with constant ω

思考与训练

analogy with linear motion with constant v

uniform linear motion: $s = vt$

displacement $s \leftrightarrow \theta$, velocity $v \leftrightarrow \omega$

for uniform circular motion, one has: $\boxed{\theta = \omega t}$

➢ time taken for one complete revolution is called **period** T

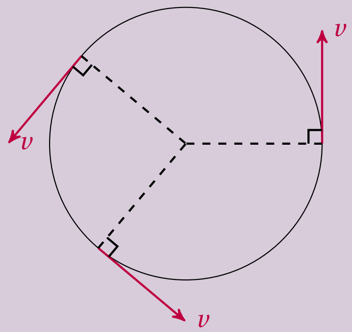
in one T, angle swiped is 2π, so $\boxed{\omega = \frac{2\pi}{T}}$

➢ uniform circular motion is still *accelerated* motion

speed is unchanged, but *velocity* is changing

direction of velocity always *tangential* to its path, so direction of velocity keeps changing

in general, any object moving along circular path is accelerating.



✍ $$\omega = \frac{2\pi}{T} = \frac{2\pi}{40} \approx 0.157 \text{ rad s}^{-1} \qquad v = \omega r = 0.157 \times 2.5 \approx 0.39 \text{ m s}^{-1}$$ □

练习 1 What is the angular velocity of the minute hand of a clock?

练习 2 A spacecraft moves around the earth in a circular orbit. The spacecraft has a speed of 7200 m s^{-1} at a height of 1300 km above the surface of the earth. Given that the radius of the earth is 6400 km. (a) What is the angular speed of this spacecraft? (b) What is its period?

1.1.4 centripetal acceleration

知识归纳与探究

centripetal acceleration is the acceleration due to the change in direction of velocity vector, it points toward the centre of circular path

consider motion along a circular path from A to B with constant speed v

under small (infinitesimal) duration of time Δt[a]

[a] A more rigorous derivation can be given by using differentiation techniques

change in velocity: $\Delta v = 2v \sin\frac{\Delta\theta}{2} \approx v\Delta\theta \quad (\text{as } \Delta\theta \to 0, \sin\Delta\theta \approx \Delta\theta)$

acceleration: $a = \frac{\Delta v}{\Delta t} \approx v\frac{\Delta\theta}{\Delta t} = v\omega \quad (\text{as } \omega = \frac{\Delta\theta}{\Delta t})$

consider motion along a circular path from A to B with constant speed v

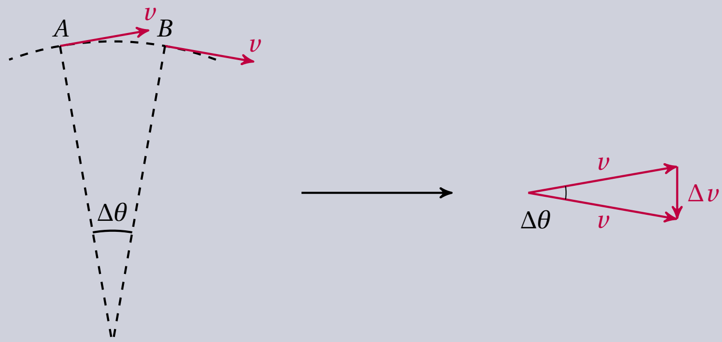
思考与解答

recall relation $v = \omega r$, we find centripetal acceleration: $\boxed{a_c = \frac{v^2}{r} = \omega^2 r}$

➢ direction of centripetal acceleration: always towards centre of circular path

➢ centripetal acceleration is only responsible for the change in *direction* of velocity

change in *magnitude* of velocity will give rise to *tangential acceleration*



this is related to *angular acceleration*[1] , which is beyond the syllabus

【练习与思考】

A racing car makes a 180° turn in 2.0 s. Assume the path is a semi-circle with a radius of 30 m and the car maintains a constant speed during the turn. (a) What is the angular velocity of the car? (b) What is the centripetal acceleration?

1.2 centripetal force

circular motion must involve change in velocity, so object is not in equilibrium

there must be a *net force* on an object performing circular motion

课前预习与思考

centripetal force (F_c) is the resultant force acting on an object

➢ moving along a circular path, and it is always directed towards centre of the circle

➢ centripetal force causes centripetal acceleration

using Newton's 2nd law: $\boxed{F_c = m\frac{v^2}{r} = m\omega^2 r}$

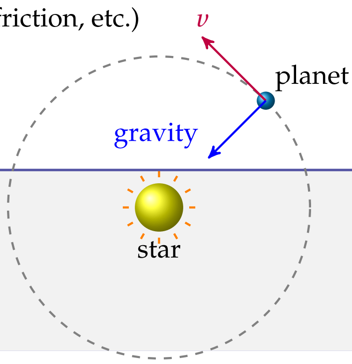
F_c is not a new force by nature, it can have a variety of origins

F_c is a resultant of forces you learned before (weight, tension, contact force, friction, etc.)

F_c acts at right angle to direction of velocity

or equivalently, if $F_{\text{net}} \perp v$ and F_{net} is of constant magnitude

then this net force provides centripetal force for circular motion



练习 3 ➢ effect of F_c: change *direction* of motion, or maintain circular orbits

to change *magnitude* of velocity, there requires a *tangential* component for the net force

[1]Angular acceleration is analogous to linear acceleration α, defined as rate of change of angular velocity: $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ $(\star)$. Similar to $v = \omega r = \frac{ds}{dt}$, the relation $a = \alpha r = \frac{dv}{dt}$ also holds.

练习 4 again the idea of tangential force is beyond the syllabus
planet orbiting around a star

gravity by the star provides centripetal force for the planet

A rock is able to orbit around the earth near the earth's surface. Let's ignore air resistance for this question, so the rock is acted by weight only. Given that radius of the earth $R = 6400$ km.

(a) What is the orbital speed of the rock? (b) What is the orbital period?

✍ weight of object provides centripetal force: $mg = \frac{mv^2}{R}$

orbital speed: $v = \sqrt{gR} = \sqrt{9.81 \times 6.4 \times 10^6} \approx 7.9 \times 10^3$ m s^{-1}

period: $T = \frac{2\pi R}{v} = \frac{2\pi \times 6.4 \times 10^6}{7.9 \times 10^3} \approx 5.1 \times 10^3$ s ≈ 85 min □

解题思路分析: A turntable can rotate freely about a vertical axis through its centre. A small object is placed on the turntable at distance $d = 40$ cm from the centre. The turntable is then set to rotate, and the angular speed of rotation is slowly increased. The coefficient of friction between the object and the turntable is $\mu = 0.30$. If the object does not slide off the turntable, find the maximum number of revolutions per minute.

✍ if object stays on turntable, friction provides the centripetal force required: $f = m\omega^2 d$

increasing ω requires greater friction to provide centripetal force

but maximum limiting friction possible is: $f_{\text{lim}} = \mu N = \mu mg$, therefore

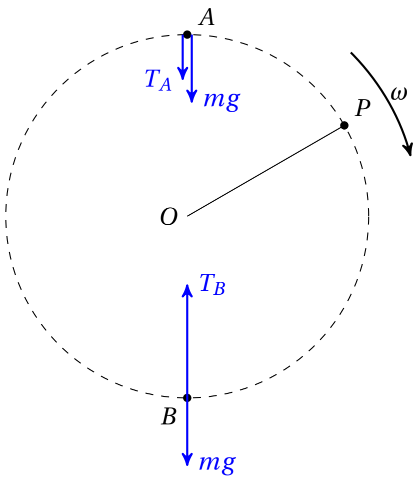
$$f \le f_{\text{lim}} \quad \Rightarrow \quad m\omega^2 d \le \mu mg \quad \Rightarrow \quad \omega^2 \le \frac{\mu g}{d} \quad \Rightarrow \quad \omega_{\max} = \sqrt{\frac{0.30 \times 9.81}{0.40}} \approx 2.71 \text{ rad s}^{-1}$$

period of revolution: $T_{\min} = \frac{2\pi}{\omega_{\max}} = \frac{2\pi}{2.71} \approx 2.32$ s

umber of revolutions in one minute: $n_{\max} = \frac{t}{T_{\min}} = \frac{60}{2.32} \approx 25.9$

【解答与反思】

Particle P of mass $m = 0.40$ kg is attached to one end of a light inextensible string of length $r = 0.80$ m. The particle is whirled at a constant angular speed ω in a vertical plane. (a) Given that the string never becomes slack, find the minimum value of ω. (b) Given instead that the string will break if the tension is greater than 20 N, find the maximum value of ω.



✍ at top of circle (point A): $F_c = T_A + mg = m\omega^2 r \quad \Rightarrow \quad T_A = m\omega^2 r - mg$

at bottom of circle (point B): $F_c = T_B - mg = m\omega^2 r \quad \Rightarrow \quad T_B = m\omega^2 r + mg$

tension is minimum at A, but string being taut requires $T \ge 0$ at any point, so $T_A \ge 0$

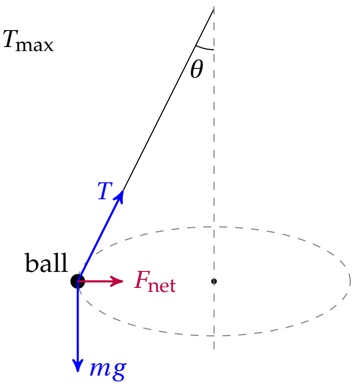
$$m\omega^2 r - mg \ge 0 \quad \Rightarrow \quad \omega^2 \ge \frac{g}{r}$$

$$\omega_{\min} = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.81}{0.80}} \approx 3.5 \text{ rad s}^{-1}$$

tension is maximum at B, but string does not break requires $T \le T_{\max}$, so $T_B \le T_{\max}$

$$m\omega^2 r + mg \le T_{\max} \quad \Rightarrow \quad \omega^2 \le \frac{T_{\max}}{m} - \frac{g}{r}$$

$$\omega_{\max} = \sqrt{\frac{T_{\max}}{m} - \frac{g}{r}} = \sqrt{\frac{20}{0.40} - \frac{9.81}{0.80}} \approx 6.1 \text{ rad s}^{-1}$$ □



✍ vertical component of tension T_y equals weight

$$T_y = mg \quad \Rightarrow \quad T\cos\theta = mg$$

$$T = \frac{mg}{\cos\theta} = \frac{0.12 \times 9.81}{\cos 25°} \approx 1.3 \text{ N}$$

net force equals horizontal component of tension T_x

so component T_x provides centripetal force

$$F_c = T_x \quad \Rightarrow \quad T\sin\theta = \frac{mv^2}{r}$$

by eliminating T and m, one can find

$$v^2 = \frac{r\tan\theta}{g} = \frac{0.10\times\tan 25°}{9.81} \quad \Rightarrow \quad v \approx 0.069\ \mathrm{m\ s^{-1}} \qquad \square$$

When the ball reaches lowest point, find its speed and the tension in the string in terms of m and l.

思考与练习

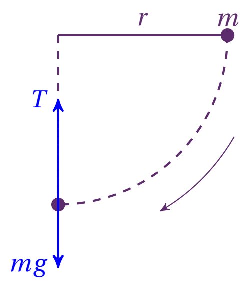
✍ energy conservation: G.P.E. loss = K.E. gain

$$mgr = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gr}$$

at lowest point: $F_c = T - mg = m\dfrac{v^2}{r}$

$$T = mg + m\frac{v^2}{r} = mg + m\frac{2gr}{r} = 3mg \qquad \square$$

Question 1.1 Suggest what provides centripetal force in the following cases. (a) An athlete running on a curved track. (b) An aeroplane banking at a constant altitude. (c) A satellite moving around the earth.

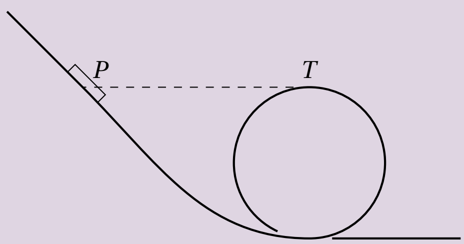


Question 1.2 A turntable that can rotate freely in a horizontal plane is covered by dry mud. When the angular speed of rotation is gradually increased, state and explain whether the mud near edge of the plate or near the mud will first leave the plate?

Question 1.3 A bucket of water is swung at a constant speed and the motion describes a circle of radius $r = 1.0$m in the vertical plane. If the water does not pour down from the bucket even when it is at the highest position, how fast do you need to swing the bucket?

练习（题 1）. Question

This question is about the design of a roller-coaster. We consider a slider that starts from rest from a point P and slides along a frictionless circular track as sketched below. P is at the same height as the top of the track T. (a) Show that the slider cannot get to T. (b) As a designer for a roller-coaster, you have to make sure the slider can reach point T and continue to slide along the track, what is the minimum height for the point of release?



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【知识点衔接】

Newton's law of gravitation states that gravitational force between two *point* masses is proportional to the product of their masses and inversely proportional to the square of their distance $\left(F_{\text{grav}} \propto \dfrac{Mm}{r^2}\right)$

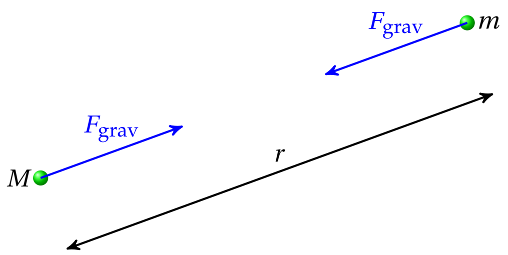
this law was formulated in *Issac Newton*'s work 'The Principia', or 'Mathematical Principles of Natural Philosophy', first published in 1687

第 2 章 Chapter 2 Gravitational Fields

2.1 gravitational forces

2.1.1 Newton's law of gravitation

any object attracts any other object through the gravitational force



gravitational attraction between M and m

> **Newton's law of gravitation** states that gravitational force between two *point* masses is proportional to the product of their masses and inversely proportional to the square of their distance $\left(F_{\text{grav}} \propto \dfrac{Mm}{r^2}\right)$

this law was formulated in *Issac Newton*'s work 'The Principia', or 'Mathematical Principles of Natural Philosophy', first published in 1687

mathematically, gravitational force takes the form: $\boxed{F_{\text{grav}} = \dfrac{GMm}{r^2}}$

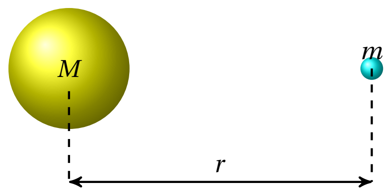
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} is the *gravitational constant*

- gravitational force is always *attractive*
- gravity is *universal*, i.e., gravitational attraction acts between *any* two masses
- Newton's law of gravitation refers to *point masses*

  i.e., particles with no size, therefore distance r can be easily defined
- a sphere with uniform mass distribution (e.g., stars, planets) can be treated as a *point model*

  distance r is taken between centres of the spheres [2]

  (see Example 2.8, field lines around a planet *seem* to point towards centre of planet)



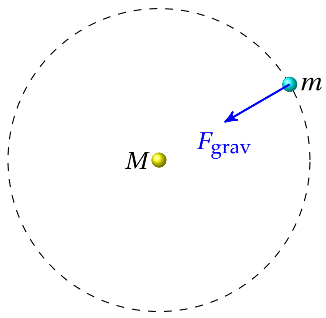
Example 2.1 The Earth can be thought as a uniform sphere of radius $R = 6.4 \times 10^6$ m and mass $M = 6.0 \times 10^{24}$ kg. Estimate the gravitational force on a man of 60 kg at sea level.

✍
$$F = \frac{GMm}{R^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 60}{(6.4 \times 10^6)^2} \approx 586 \text{ N}$$
□

Question 2.1 Estimate the gravitational force between you and your deskmate.

[2]This is known as *shell theorem*: a spherically symmetric shell (i.e., a hollow ball) affects external objects gravitationally as though all of its mass were concentrated at its centre, and it exerts no net gravitational force on any object inside, regardless of the object's location within the shell. (⋆)

2.1.2 planetary motion



a planet/satellite orbiting around a star/earth

a planet/satellite can move around a star/earth in circular orbit

circular motion requires centripetal force

for these objects, gravitational force provides centripetal force

$$F_{\text{grav}} = F_c \quad \Rightarrow \quad \boxed{\frac{GMm}{r^2} = \frac{mv^2}{r}} \quad \text{or} \quad \boxed{\frac{GMm}{r^2} = m\omega^2 r}$$

Example 2.2 GPS (Global Positioning System) satellites move in a circular orbits at about 20000 km above the earth's surface. The Earth has a radius $R = 6.4 \times 10^6$ m and mass $M = 6.0 \times 10^{24}$ kg. (a) Find the speed of GPS satellites. (b) Find its orbital period.

✍
$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \Rightarrow \quad v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{6.4 \times 10^6 + 2.0 \times 10^7}} \approx 3.9 \times 10^3 \text{ m s}^{-1}$$

$$v = \frac{2\pi r}{T} \quad \Rightarrow \quad T = \frac{2\pi r}{v} = \frac{2\pi \times (6.4 \times 10^6 + 2.0 \times 10^7)}{3.9 \times 10^3} \approx 4.3 \times 10^4 \text{ s} \approx 11.8 \text{ hours}$$ □

Example 2.3 A **geostationary satellite** moves in a circular orbit that appears motionless to ground observers. The satellite follows the Earth's rotation, so the satellite rotates from west to east above equator with an orbital period of 24 hours. Find the radius of this orbit.

✍
$$\frac{GMm}{r^2} = m\omega^2 r \quad \Rightarrow \quad \frac{GMm}{r^2} = m\left(\frac{2\pi}{T}\right)^2 r \quad \Rightarrow \quad r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left(\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times (24 \times 3600)^2}{4\pi^2}\right)^{1/3} \approx 4.23 \times 10^7 \text{ m}$$ □

Example 2.4 Assuming the planets in the solar system all move around the sun in circular orbits, show that the square of orbital period is proportional to the cube of orbital radius. [3]

✍
$$\frac{GMm}{r^2} = m\omega^2 r \quad \Rightarrow \quad \frac{GMm}{r^2} = m\left(\frac{2\pi}{T}\right)^2 r \quad \Rightarrow \quad T^2 = \frac{4\pi^2}{GM} \cdot r^3$$

G is gravitational constant, M is mass of the sun, so $\frac{4\pi^2}{GM}$ is a constant, so $T^2 \propto r^3$ □

Question 2.2 Given that it takes about 8.0 minutes for light to travel from the sun to the earth. (a) What is the mass of the sun? (b) At what speed does the earth move around the sun?

2.1.3 apparent weight

an object's *actual weight* is the gravitational attraction exerted by the earth's gravity

an object's *apparent weight* is the upward force (e.g., normal contact force exerted by ground, tension in a spring balance, etc.) that opposes gravity and prevents the object from falling

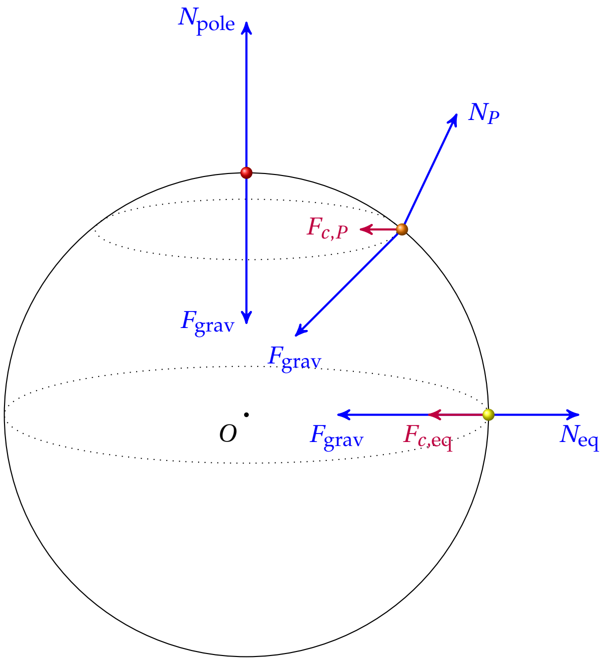
apparent weight can be different from actual weight due to vertical acceleration or buoyancy

but if we consider rotation of the earth, this also causes apparent weight to be lessened

---

[3] This is known as *Kepler's 3rd law* for planetary motions. In the early 17th century, German astronomer Johannes Kepler discovered three scientific laws which describes how planets move around the sun. This $T^2 \propto r^3$ relation not only holds for circular orbits but are also correct for elliptical orbits.

Isaac Newton proved that Kepler's laws are consequences of his own law of universal gravitation, and therefore explained why the planets move in this way. (⋆)



apparent weight at various positions near earth's surface (not to scale)

object resting on ground is actually rotating together with earth

resultant of gravitational force and contact force should provide centripetal force

for object on equator: $F_{c,\text{eq}} = m\omega^2 R \quad \Rightarrow \quad F_{\text{grav}} - N_{\text{eq}} = m\omega^2 R \quad \Rightarrow \quad N_{\text{eq}} = \dfrac{GMm}{R^2} - m\omega^2 R$

for object at poles: $F_{c,\text{pole}} = 0 \quad \Rightarrow \quad F_{\text{grav}} - N_{\text{pole}} = 0 \quad \Rightarrow \quad N_{\text{pole}} = \dfrac{GMm}{R^2}$

at lower latitudes, object describe larger circles, hence requires greater centripetal force

this offsets the balancing normal force, so apparent weight decreases near the equator

Example 2.5 A stone of mass 5.0 kg is hung from a newton-meter near the equator. The Earth can be considered to be a uniform sphere of radius $R = 6370$ km and mass $M = 5.97 \times 10^{24}$ kg. (a) What is the gravitational force on the stone? (b) What is the reading on the meter?

✍ gravitational force: $F_{\text{grav}} = \dfrac{GMm}{R^2} = \dfrac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 5.0}{(6.37 \times 10^6)^2} \approx 49.07$ N

centripetal force required: $F_c = m\omega^2 R = m\left(\dfrac{2\pi}{T}\right)^2 R = 5.0 \times \dfrac{4\pi^2}{(24 \times 3600)^2} \times 6.37 \times 10^6 \approx 0.17$ N

apparent weight, or reading on meter: $N = F_{\text{grav}} - F_c = 49.07 - 0.17 \approx 48.90$ N □

Question 2.3 Why astronauts in space stations are said to be *weightless*?

Question 2.4 How do you find the apparent weight of an object at an arbitrary latitude P? Does the apparent weight act vertically downwards? Give your reasons.

2.2 gravitational fields

to explain how objects exert gravitational attraction upon one another at a distance, we introduce the concept of *force fields*

gravitational field is a region of space where a mass is acted by a force

any mass M (or several masses) can produce a gravitational field around it

a test mass m within this field will experience a gravitational force

to describe the effect on a small mass m in the field, we will further introduce

- *gravitational field strength*, to help us compute gravitational force on objects

- *gravitational potential*, to help us compute gravitational potential energy between objects

2.3 gravitational field strength

2.3.1 gravitational field strength

gravitational field strength is defined as gravitational force per unit mass: $\boxed{g = \dfrac{F_{\text{grav}}}{m}}$

- unit of g: $[g] = \text{N kg}^{-1} = \text{ m s}^{-2}$, same unit as acceleration
- field strength due to an isolated source of mass M

  at distance r from the source, a test mass m is acted by a force: $F_{\text{grav}} = \dfrac{GMm}{r^2}$

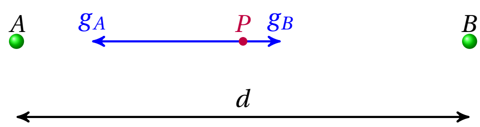
  field strength at this position: $g = \dfrac{F_{\text{grav}}}{m} = \quad \Rightarrow \quad \boxed{g = \dfrac{GM}{r^2}}$

  note that the field is produced by the source M, so field strength g depends on M, not m
- field strength g is a *vector* quantity, it has a direction

  gravitation is *attractive*, so g points towards source mass

  to compute combined field strength due to several sources, should perform *vector sum* of contributions from each individual

Example 2.6 Star A of mass 6.0×10^{30} kg and star B of mass 1.5×10^{30} kg are separated by a distance of 2.0×10^{12} m. (a) What is the field strength at the mid-point P of the two stars? (b) If a comet of mass 4.0×10^{6} kg is at the mid-point, what force does it experience?



g_A acts towards A, g_B acts towards B, they are in opposite directions

$$g_P = g_A - g_B = \frac{GM_A}{r_A^2} - \frac{GM_B}{r_B^2} = 6.67 \times 10^{-11} \times \left[\frac{6.0 \times 10^{30}}{(1.0 \times 10^{12})^2} - \frac{1.5 \times 10^{30}}{(1.0 \times 10^{12})^2} \right] \approx 3.0 \times 10^{-4} \text{ N kg}^{-1}$$

force on comet: $F = mg = 4.0 \times 10^{6} \times 3.0 \times 10^{-4} \approx 1.2 \times 10^{3}$ N □

2.3.2 acceleration of free fall

if field strength g is known, gravitational force on an object of mass m is: $F_{\text{grav}} = mg$

if the object is acted by gravity only, then $F_{\text{net}} = F_{\text{grav}} \quad \Rightarrow \quad ma = mg \quad \Rightarrow \quad \boxed{a = g}$ [4]

this shows gravitational field strength gives the acceleration of free fall!

Example 2.7 The earth has a radius of 6370 km. (a) Find the mass of the earth. [5] (b) Find the acceleration of free fall at the top of Mount Everest. (height of Mount Everest $H \approx 8.8$ km)

[4] Rigorously speaking, the two m's are different concepts. There is the *inertia* mass, decribing how much an object resists the change of state of motion. There is also the *gravitational* mass, describing the effect produced and experienced by the object in gravitational fields. Yet no experiment has ever demonstrated any significant difference between the two. The reason why the two masses are identical is very profound. We have shown here acceleration of free fall equals gravitational field strength, but Albert Einstein's *equivalence principle* suggests that it is actually impossible to distinguish between a uniform acceleration and a uniform gravitational field. This idea lies at the heart of the *general theory of relativity*, where I should probably stop going further.

[5] British scientist Henry Cavendish devised an experiment in 1798 to measure the gravitational force between masses in his laboratory. He was the first man to yield accurate values for the gravitational constant G. Then he was able to carry out this calculation, referred by himself as 'weighing the world'.

✍ consider acceleration of free fall near surface of earth:

$$g_s = \frac{GM}{R^2} \quad \Rightarrow \quad 9.81 = \frac{6.67\times10^{-11}\times M}{(6.37\times10^6)^2} \quad \Rightarrow \quad M \approx 5.97\times10^{24}\ \mathrm{kg}$$

at top of Mount Everest:

$$g_{\mathrm{ME}} = \frac{GM}{(R+H)^2} = \frac{6.67\times10^{-11}\times5.97\times10^{24}}{(6.37\times10^6+8.8\times10^3)^2} \approx 9.78\ \mathrm{N\ kg^{-1}} \quad \Rightarrow \quad a_{\mathrm{ME}} \approx 9.78\ \mathrm{m\ s^{-2}}$$ □

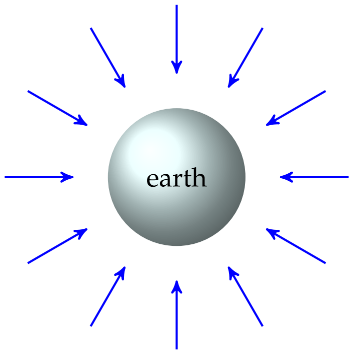
2.3.3 gravitational field lines

gravitational field lines are drawn to graphically represent the pattern of field strength

- *direction* of field lines show the *direction* of field strength in the field
- *spacing* between field lines indicates the *strength* of the gravitational field
- gravitational field lines always end up at a mass

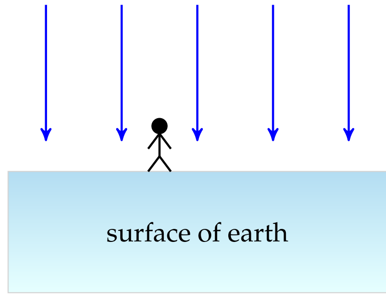
  this arises from the attractive nature of gravitation

Example 2.8 field around the earth



radial field (field lines normal to surface)

Example 2.9 field near earth's surface



almost a *uniform* field
(field lines are parallel and equally spaced)

2.4 gravitational potential & potential energy

2.4.1 potential energy

potential energy is the energy possessed by an object due to its position in a force field
work done *by* force field decreases P.E., and work done *against* a force field increases P.E.
let W be work by the force field, then we have: $\boxed{W = -\Delta E_p}$

to define potential energy of an object at a specific point X, we can

(1) choose a position where potential energy is defined to be zero

(2) find work done by force field to bring the object from zero P.E. point to X

(3) consider change in P.E.: $\Delta E_p = E_{p,X} - E_{p,\text{initial}} = E_{p,X} - 0 = E_{p,X}$

but $\Delta E_p = -W$, so P.E. at point X is found: $E_{p,X} = -W$

so potential energy is equal to (negative) work done to move the object to a specific position

gravitational potential energy near earth's surface

we may choose a zero G.P.E. point, for example, $E_p(0) = 0$ at sea level

if mass m is moved up for a height h, work done by gravity is $W = -mgh$[6]

this causes a change in gravitational potential energy $\Delta E_p = -W = mgh$

then at altitude h, G.P.E. can be given by $E_p(h) = mgh$

2.4.2 gravitational potential energy

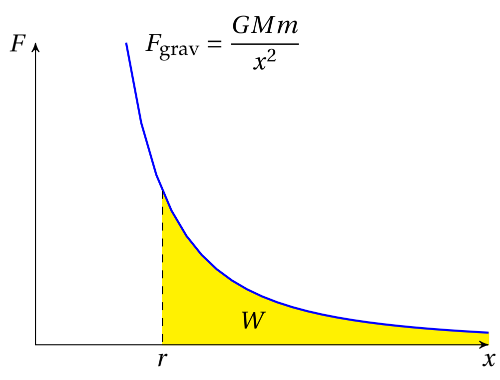
we are now ready to derive an expression for G.P.E. between two masses M and m

we define $E_p = 0$ at $r = \infty$ (choice of zero potential energy, no force so no G.P.E.), then

> **gravitation potential energy** is equal to the work done by gravitational force to bring a mass to a specific position from *infinity*

consider a mass m at infinity with zero energy and a source mass M at origin

let's find out how much work is done by gravitational force to pull m towards the origin



but F_{grav} varies as inverse square of separation x

so here we need to evaluate work done by a non-constant force

we can plot a F-x graph, then magnitude of work done equals area under the graph

integrate[7] to evaluate the area: $W = \int_r^\infty \frac{GMm}{x^2}\,\mathrm{d}x = -\frac{GMm}{x}\bigg|_r^\infty = \frac{GMm}{r}$

[6]Negative sign because this is actually work against gravity.

[7]In general, work done by a non-constant force over large distance is $W = \int_{\text{initial}}^{\text{final}} F\,\mathrm{d}x$.

For our case, x is the displacement away from the source, but gravitational force tends to pull the mass towards the source. F and x are in opposite directions, a negative sign is needed for F. Therefore the work done by gravity to bring mass m from infinity is: $W = \int_\infty^r F\,\mathrm{d}x = \int_\infty^r \left(-\frac{GMm}{x^2}\right)\mathrm{d}x = +\frac{GMm}{x}\bigg|_\infty^r = \frac{GMm}{r}$.

$$\Delta E_p = -W \quad \Rightarrow \quad E_p(r) - E_p(\infty) = -\frac{GMm}{r}$$

but we have defined $E_p(\infty) = 0$, therefore: $\boxed{E_p(r) = -\frac{GMm}{r}}$

$E_p(r)$ gives G.P.E between masses M and m when they are at distance r from each other

- as $r \to \infty$, $E_p \to 0$, this agrees with our choice of zero G.P.E. point
- potential energy is a *scalar* quantity (negative sign cannot be dropped)
- G.P.E. is always *negative*, this is due to *attractive* nature of gravity
  to separate masses, work must be done to overcome attraction
  so G.P.E. increases with separation r, i.e., G.P.E. is maximum at infinity, which is zero
  G.P.E. between masses at finite separation must be less than zero
- $E_p = mgh$ is only applicable near earth's surface where field is almost *uniform*
  $E_p = -\frac{GMm}{r}$ is a more *general* formula for gravitational potential energy [8]

Example 2.10 A meteor is travelling towards a planet of mass M. When it is at a distance of r_1 from centre of M, it moves at speed v_1. When it is r_2 from M, it moves at speed v_2. Assume only gravitational force applies, establish a relationship between these quantities.

✍ energy conservation: K.E. + G.P.E. = const $\Rightarrow \frac{1}{2}mv_1^2 + \left(-\frac{GMm}{r_1}\right) = \frac{1}{2}mv_2^2 + \left(-\frac{GMm}{r_2}\right)$ □

Example 2.11 If an object is thrown from the surface of a planet at sufficiently high speed, it might escape from the influence of the planet's gravitational field. The minimum speed required is called the *escape velocity*. Using the data from previous examples, find the escape velocity from the surface of earth.

✍ assuming no energy loss to air resistance, then total energy is conserved

$$\text{K.E.} + \text{G.P.E. at surface of planet} = \text{K.E.} + \text{G.P.E. at infinity}$$

$$\frac{1}{2}mu^2 + \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv^2 + 0 \quad \xRightarrow{v \geq 0} \quad u^2 \geq \frac{2GM}{R} \quad \Rightarrow \quad u_{\min} = \sqrt{\frac{2GM}{R}}$$

for earth, escape velocity $u_{\min} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{6.4 \times 10^6}} \approx 1.12 \times 10^4 \text{ m s}^{-1}$ □

Question 2.5 A planet of uniform density distribution is of radius R and mass M. A rock falls from a height of $3R$ above the surface of the planet. Assume the planet has no atmosphere, show that the speed of the rock when it hits the ground is $v = \sqrt{\frac{3GM}{4R}}$.

Question 2.6 A space probe is travelling around a planet of mass M in a circular orbit of radius r. (a) Show that the total mechanical energy (sum of kinetic energy and gravitational energy) of the space probe is $E_{\text{total}} = -\frac{2GMm}{r}$. (b) If the space probe is subject to small resistive forces, state the change to its orbital radius and its orbiting speed.

Question 2.7 A *black hole* is a region of spacetime where gravitation is so strong that even light can escape from it. For a star of mass M to collapse and form a black hole, it has to be compressed below a certain radius. (a) Show that this radius is given by $R_S = \frac{2GM}{c^2}$, known as the *Schwarzschild radius*[9]. (b) Show that the Schwarzschild radius for the sun is about 3 km.

---

[8] One can recover $\Delta E_p = mg\Delta h$ from $E_p = -\frac{GMm}{r}$. Near the earth's surface, if $r_1 \approx r_2 \approx R$, and $r_1 > r_2$, then we have: $\Delta E_p = E_p(r_1) - E_p(r_2) = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = GMm\frac{r_1 - r_2}{r_1 r_2} \approx m\frac{GM}{R^2}\Delta r \xlongequal{g=GM/R^2} mg\Delta h$.

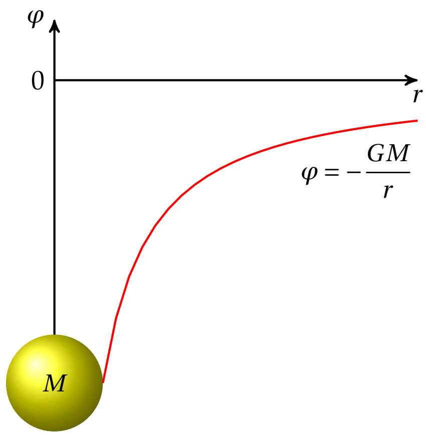
[9] When you deal with very strong gravitational fields, Newton's law of gravitation breaks down and effects of Einstein's *general theory of relativity* come into play. The radius of a *Newtonian* black hole being equal to the radius of a Schwarzschild black hole is a mere coincidence.

2.4.3 gravitational potential

it is useful to introduce a quantity called *potential* at a specific point in a gravitational field

gravitational potential can be considered as the potential energy per unit mass: $\varphi = \dfrac{E_p}{m}$

gravitational potential at a point is defined as the work done to bring *unit* mass from *infinity* to that point



- unit: $[\varphi] = \mathrm{J\,kg^{-1}}$
- gravitational potential due to an isolated source M

$$\varphi = \frac{E_p}{m} = \frac{-\frac{GMm}{r}}{m} \quad \Rightarrow \quad \boxed{\varphi = -\frac{GM}{r}}$$

- potential at infinity is zero: $\varphi_\infty = 0$

  this is our choice of zero potential point
- gravitational potential is a *scalar*

  combined potential due to several masses equals scalar sum of potential of each individual
- gravitational potential is always *negative*

  again this arises from attractive nature of gravity

  work is done to pull unit mass away from source

  farther from source means higher potential

Example 2.12 A star A of mass $M_A = 1.5 \times 10^{30}$ kg and a planet B of mass $M_B = 6.0 \times 10^{26}$ kg form an isolated astronomical system. Point P is between A and B, and is at distance $r_A = 2.0 \times 10^{12}$ m from A, and distance $r_B = 8.0 \times 10^{10}$ m from B. (a) Find the gravitational potential at P. (b) A meteor is initially at very large distance from the system with negligible speed. It then travels towards point P due to the gravitational attraction. Find its speed when it reaches P.

✍ gravitational potential at P: $\varphi_P = \varphi_A + \varphi_B = \left(-\dfrac{GM_A}{r_A}\right) + \left(-\dfrac{GM_B}{r_B}\right)$

$$\varphi_P = -6.67 \times 10^{-11} \times \left(\frac{1.5 \times 10^{30}}{2.0 \times 10^{12}} + \frac{6.0 \times 10^{26}}{8.0 \times 10^{10}}\right) \approx -5.05 \times 10^7 \ \mathrm{J\,kg^{-1}}$$

gain in K.E. = loss in G.P.E.: $\dfrac{1}{2}mv^2 = m\Delta\varphi \quad \Rightarrow \quad v^2 = 2(\varphi_\infty - \varphi_P) = -2\varphi_P$

$$v = \sqrt{-2 \times (-5.05 \times 10^7)} \approx 1.01 \times 10^4 \ \mathrm{m\,s^{-1}}$$ □

Example 2.13 The Moon may be considered to be an isolated sphere of radius $R = 1.74 \times 10^3$ km. The gravitational potential at the surface of the moon is about $-2.82 \times 10^6\ \mathrm{J\,kg^{-1}}$. (a) Find the mass of the moon. (b) A stone travels towards the moon such that its distance from the centre of the moon changes from $3R$ to $2R$. Determine the change in gravitational potential. (c) If the stone starts from rest, find its final speed.

✍ at surface: $\varphi(R) = -\dfrac{GM}{R} \quad \Rightarrow \quad -2.82 \times 10^6 = -\dfrac{6.67 \times 10^{-11} \times M}{1.74 \times 10^6} \quad \Rightarrow \quad M = 7.36 \times 10^{22}$ kg

from $3R$ to $2R$: $\Delta\varphi = \varphi_{(3R)} - \varphi_{(2R)} = \left(-\dfrac{GM}{3R}\right) - \left(-\dfrac{GM}{2R}\right) = \dfrac{GM}{6R} = \dfrac{2.82 \times 10^6}{6} \approx 4.70 \times 10^5\ \mathrm{J\,kg^{-1}}$

note this change is a *decrease* in gravitational potential

gain in K.E. = loss in G.P.E.: $\dfrac{1}{2}mv^2 = m\Delta\varphi \quad \Rightarrow \quad v = \sqrt{2\Delta\varphi} = \sqrt{2 \times 4.70 \times 10^5} \approx 970\ \mathrm{m\,s^{-1}}$ □

Question 2.8 Given that the moon is of radius 1700 km and mass 7.4×10^{22} kg. (a) Find the change in gravitational potential when an object is moved from moon's surface to 800 km above the surface. (b) If a rock is projected vertically upwards with an initial speed of 1800 m s^{-1} from surface, find the rock's speed when it reaches a height of 800 km. (c) Suggest whether the rock can escape from the moon's gravitational field completely.

第 3 章 Chapter 3 Oscillation

3.1 oscillatory motion

oscillation refers to a repetitive back and forth motion about its *equilibrium position*

the equilibrium position is a point where all forces on oscillator are balanced
release an object from its equilibrium position from rest, it will stay at rest
examples of oscillation includes pendulum of a clock, vibrating string, swing, etc.

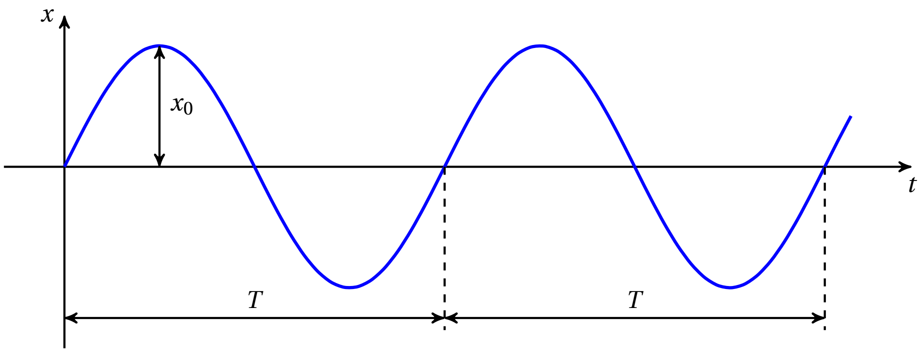
3.1.1 amplitude, period, frequency

to describe motion of an oscillator, we define the following quantities:

- **displacement** (x): distance from the equilibrium position
- **amplitude** (x_0): maximum displacement from the equilibrium position
- **period** (T): time for one complete oscillation
- **frequency** (f): number of oscillations per unit time

frequency is related to period as: $\boxed{f = \frac{1}{T}}$

displacement x varies with time t repetitively, for which we can plot an x-t graph
amplitude x_0 and period T are labelled on the graph



displacement-time graph for a typical oscillator

3.1.2 phase angle

the point that an oscillator has reached within a complete cycle is called **phase angle** (ϕ)

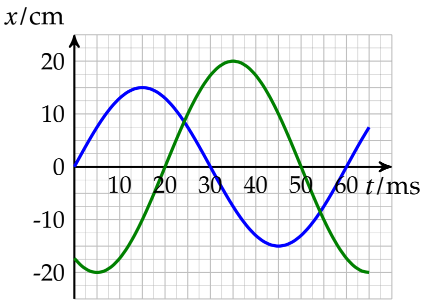
- unit of phase angle: $[\phi] = \text{rad}$

it looks like an angle, but better think of it as a number telling fraction of a complete cycle

- we use **phase difference** $\Delta\phi$ to compare how much one oscillator is ahead of another

$\Delta\phi$ is found in terms of fraction of an oscillation: $\Delta\phi = \frac{\Delta t}{T} \times 2\pi$ (also measured in radians)

Example 3.1 Compare the two oscillations from the x-t graph below.



both have period $T = 60$ ms

frequency $f = \dfrac{1}{60 \times 10^{-3}} \approx 16.7$ Hz

they are of different amplitudes

one has $x_0 = 15$ cm, the other has $x_0 = 20$ cm

time difference: $\Delta t = 20$ ms

phase difference: $\Delta\phi = \dfrac{\Delta t}{T} \times 2\pi = \dfrac{20}{60} \times 2\pi = \dfrac{2\pi}{3}$ rad

3.1.3 acceleration & restoring force

for any oscillatory motion, consider its velocity and acceleration at various positions

its acceleration must be always pointing towards the equilibrium position

resultant force always acts in the direction to restore the system back to its equilibrium point, this net force is known as the **restoring force**

if at equilibrium position, then no acceleration or restoring force

3.2 simple harmonic oscillation

> if an oscillator has an acceleration always proportional to its displacement from the equilibrium position, and acceleration is in opposite direction to displacement, then the oscillator is performing **simple harmonic motion**

many phenomena can be approximated by simple harmonics

examples are motion of a pendulum, molecular vibrations, etc.

complicated motions can be decomposed into a set of simple harmonics

simple harmonic motion provides a basis for the study of many complicated motions [10]

3.2.1 equation of motion

defining equation for simple harmonics can be written as $\boxed{a = -\omega^2 x}$

ω is some constant, so a is proportional to x

the minus sign shows a and x are in opposite directions

general solution to this this equation of motion[11] takes the form: $\boxed{x = x_0 \sin(\omega t + \phi)}$

x_0 represents the amplitude, ω is called the angular frequency, ϕ is the phase angle

angular frequency

- **angular frequency** satisfies the relation: $\boxed{\omega = \dfrac{2\pi}{T} = 2\pi f}$
- unit of angular frequency: $[\omega] = \text{rad} \cdot \text{s}^{-1}$
- angular frequency ω is determined by the system's *physical constants* only

  if an object is set to oscillate *freely* with no external force, its period will always be the same

  frequency of an free oscillatory system is called the **natural frequency**

---

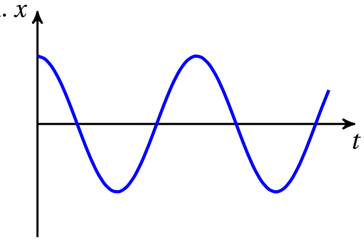
[10]This can be done through a mathematical technique known as *Fourier analysis*. For example, a uniform circular motion can be considered as the combination of two simple harmonic motion in x- and y-directions.

[11]You probably know that acceleration can be written as the second derivative of displacement: $a = \dfrac{\mathrm{d}^2 x}{\mathrm{d}t^2}$, so $a = -\omega^2 x$ is equivalent to $\dfrac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \omega^2 x = 0$, which a *second-order differential equation*. If you do not know how to solve it, you may have the chance to study this in an advanced calculus course.

phase angle

- phase angle ϕ is dependent on *initial conditions* (e.g. initial position and initial speed at $t = 0$?)
- in many cases, phase angle term can be avoided if a suitable trigonometric function is chosen

Example 3.2 A simple harmonic oscillator is displaced by 6.0 cm from its rest position and let go at $t = 0$. Given that the period of this system is 0.80 s, state an equation for its displacement-time relation.



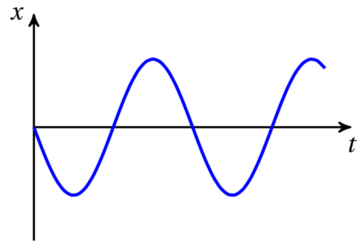
✍ angular frequency: $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.80} = \dfrac{5\pi}{2}\pi$ rad s^{-1}

initial displacement $x(0) = +x_0 = 6.0$ cm

for displacement-time relation, we use cosine function

$$x(t) = x_0 \cos \omega t \quad \Rightarrow \quad x = 6.0 \cos\left(\frac{5\pi}{2} t\right) \qquad \square$$

Example 3.3 A simple harmonic oscillator is initially at rest. At $t = 0$, it is given an initial speed in the negative direction. Given that the frequency is 1.5 Hz and the amplitude is 5.0 cm, state an equation for its displacement-time relation.



✍ angular frequency: $\omega = 2\pi f = 2\pi \times 1.5 = 3\pi$ rad s^{-1}

initial displacement $x(0) = 0$

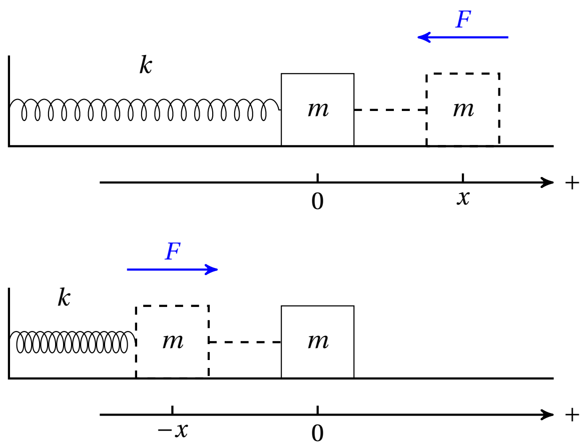
for displacement-time relation, we use sine function

$$x(t) = -x_0 \cos \omega t \quad \Rightarrow \quad x = -5.0 \sin(3\pi t) \qquad \square$$

3.2.2 examples of simple harmonics

mass-spring oscillator

a mass-spring oscillator system consists of a block of mass m and an ideal spring



restoring force acting on the ideal mass-spring oscillator

when a spring is stretched or compressed by a mass, the spring develops a restoring force

magnitude of this force obeys *Hooke's law*: $F = kx$

direction of this force is in opposite direction to displacement x

take vector nature of force into account, we find

$$F_{\text{net}} = ma \quad \Rightarrow \quad -kx = ma \quad \Rightarrow \quad a = -\frac{k}{m} x$$

spring constant k and mass m are constants, so $a \propto x$

negative sign shows a and x are in opposite directions

so mass-spring oscillator executes simple harmonic motion

compare with $a = -\omega^2 x \quad \Rightarrow \quad \omega^2 = \dfrac{k}{m} \quad \Rightarrow \quad \boxed{\omega = \sqrt{\dfrac{k}{m}}}$

period of mass-spring oscillator: $T = \dfrac{2\pi}{\omega} \quad \Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{k}}$

➢ period and frequency are solely determined by mass of oscillator m and spring constant k

identical mass-spring systems will oscillate at same frequency no matter what amplitude

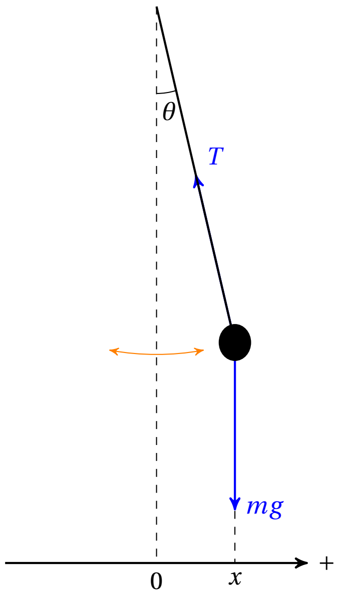
➢ $m \nearrow \Rightarrow T \nearrow$, greater mass means greater inertia, oscillation becomes slower

➢ $k \nearrow \Rightarrow T \searrow$, greater k means stiffer spring, greater restoring force makes oscillation go faster

simple pendulum

a simple pendulum is set up by hanging a bob on a light cord from a fixed point

displace the bob by some angle and release from rest, it can swing freely



one can show this performs simple harmonic motion for *small-angle* oscillation

if angular displacement θ is small, then the pendulum has almost no vertical displacement, the motion can be considered to be purely horizontal

vertically: $T\cos\theta \approx mg \xRightarrow{\cos\theta\approx 1 \text{ as } \theta\to 0} T \approx mg$

horizontally: $-T\sin\theta = ma \xRightarrow{\sin\theta = x/L} a \approx -\dfrac{g}{L}x$

this shows simple pendulum undergoes simple harmonics

compare with defining equation for simple harmonics:

$$a = -\omega^2 x \quad \Rightarrow \quad \omega = \sqrt{\frac{g}{L}}$$

period for a simple pendulum: $\boxed{T = 2\pi\sqrt{\dfrac{L}{g}}}$

➢ period and frequency of a pendulum are determined by length of the string L only

as long as angular displacement remains small, frequency does not depend on amplitude

fix length L, then simple pendulum oscillates at same frequency no matter what amplitude

➢ $L \nearrow \Rightarrow T \nearrow$, longer pendulums oscillate more slowly

➢ $g \searrow \Rightarrow T \nearrow$, if there is no gravity ($g = 0$), then the bob will not move at all ($T \to \infty$)

Question 3.1 A cylindrical tube of total mass m and cross sectional area A floats upright in a liquid of density ρ. When the tube is given a small vertical displacement and released, the magnitude of the resultant force acting on the tube is related to its vertical displacement y by the expression: $F_{\text{net}} = \rho g A y$. (a) Show that the tube executes simple harmonic motion. (b) Find an expression for the frequency of the oscillation.

Question 3.2 A small glider moves along a horizontal air track and bounces off the buffers at the ends of the track. Assume the track is frictionless and the buffers are perfectly elastic, state and explain whether the glider describes simple harmonic motion.

3.2.3 velocity & acceleration

displacement of simple harmonic oscillator varies with time as: $x = x_0\sin(\omega t + \phi)$

from this displacement-time relation, we can find velocity and acceleration relations

velocity

to find velocity-time relation, let's recall that velocity v is rate of change of displacement x

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}x_0\sin(\omega t + \phi) \quad \Rightarrow \quad \boxed{v(t) = \omega x_0\cos(\omega t + \phi)}$$

by taking $v^2 + \omega^2 x^2$, the sine and cosine terms can be eliminated, we find:

$$v^2 + \omega^2 x^2 = \omega^2 x_0^2\cos^2(\cdots) + \omega^2 x_0^2\sin^2(\cdots) = \omega^2 x_0^2$$

this gives velocity-displacement relation: $\boxed{v(x) = \pm\omega\sqrt{x_0^2 - x^2}}$

➢ at equilibrium position $x = 0$, speed is maximum: $\boxed{v_{\max} = \omega x_0}$

➢ when $x = \pm x_0$, oscillator is momentarily at rest: $v = 0$

acceleration

acceleration-time relation is found by further taking rate of change of velocity v

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\omega x_0 \cos(\omega t + \phi) \quad \Rightarrow \quad \boxed{a(t) = -\omega^2 x_0 \sin(\omega t + \phi)}$$

this is actually unnecessary, if we compare this with $x(t) = x_0 \sin(\omega t + \phi)$, we have: $a = -\omega^2 x$

we have recovered the definition for simple harmonics

(if $a \propto x$ and in opposite directions to x, then simple harmonic motion)

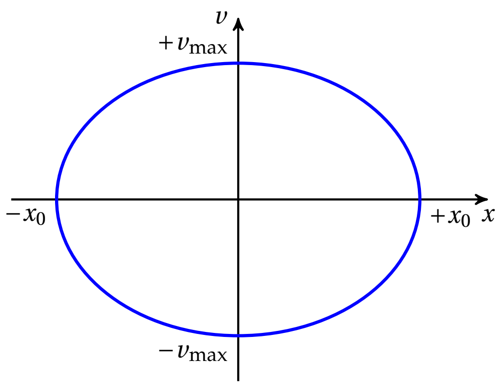
so acceleration-displacement relation is given by the defining equation explicitly $\boxed{a(x) = -\omega^2 x}$

➢ at equilibrium position $x = 0$, zero acceleration

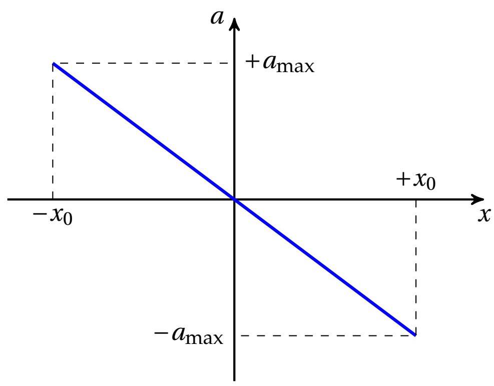
➢ when $x = \pm x_0$, acceleration is greatest: $\boxed{a_{\max} = \omega^2 x_0}$

let's take $x = x_0 \sin\omega t$ as example, changes of x, v, a over time are listed below

time t	0	$\frac{1}{4}T$	$\frac{1}{2}T$	$\frac{3}{4}T$	T
displacement: $x = x_0 \sin\omega t$	0	+max	0	−max	0
velocity: $v = \omega x_0 \cos\omega t$	+max	0	−max	0	+max
acceleration: $a = -\omega^2 x = -\omega^2 x_0 \sin\omega t$	0	−max	0	+max	0



velocity-displacement graph



acceleration-displacement graph

Example 3.4 The motion of a simple pendulum is approximately simple harmonic. As the pendulum swings from one side to the other end, it moves through a distance of 6.0 cm and the time taken is 1.0 s. (a) State the period and amplitude. (b) Find the greatest speed during the oscillation. (c) Find its speed when displacement $x = 1.2$ cm.

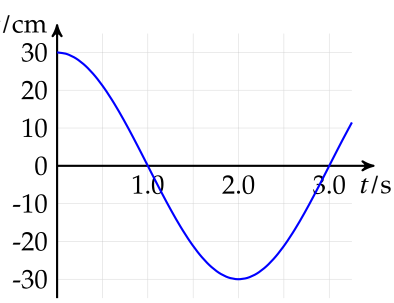
✍ period: $T = 2 \times 1.0 = 2.0$ s, and amplitude: $x_0 = \frac{1}{2} \times 6.0 = 3.0$ cm

angular frequency: $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.0} = \pi$ rad s^{-1}

greatest speed: $v_{\max} = \omega x_0 = \pi \times 3.0 \approx 9.4$ cm s^{-1}

speed at 1.2 cm: $v = \omega\sqrt{x_0^2 - x^2} = \pi \times \sqrt{3.0^2 - 1.2^2} \approx 8.6$ cm s^{-1} □

Example 3.5 Given the x-t graph of a simple harmonic oscillator. (a) Find its speed at $t = 0$. (b) Find its greatest speed. (b) Find its acceleration at $t = 1.0$ s.



✍ at $t = 0$, $x = +x_0 \quad \Rightarrow \quad v = 0$ (zero gradient)

from graph: amplitude $x_0 = 30$ cm, period $T = 4.0$ s

angular frequency: $\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$ rad s^{-1}

greatest speed: $v_{\max} = \omega A = \frac{\pi}{2} \times 30 \approx 47$ cm s^{-1}

at $t = 1.0$ s, $x = 0 \quad \Rightarrow \quad a = 0$

(equilibrium position so no acceleration) □

Question 3.3 Assume the motion of a car engine piston is simple harmonic. The piston completes 3000 oscillations per minute. The amplitude of the oscillation is 4.0 cm. (a) Find the greatest speed. (b) Find the greatest acceleration.

3.2.4 vibrational energy

consider the *ideal* mass-spring oscillator, its vibrational energy consists of two parts:

- kinetic energy of the mass: $E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t \xlongequal{v=\pm\omega\sqrt{x_0^2-x^2}} \frac{1}{2}m\omega^2(x_0^2 - x^2)$
- (elastic) potential energy in the spring: $E_p = \frac{1}{2}kx^2 \xlongequal{\omega=\sqrt{\frac{k}{m}}} \frac{1}{2}m\omega^2 x^2$

total energy of the oscillator: $E = E_k + E_p \quad \Rightarrow \quad \boxed{E = \frac{1}{2}m\omega^2 x_0^2}$

➢ although this formula is derived from the mass-spring model

$E = \frac{1}{2}m\omega^2 x_0^2$ can be used to compute vibrational energy of all simple harmonic oscillators

➢ for an ideal system, total energy remains constant

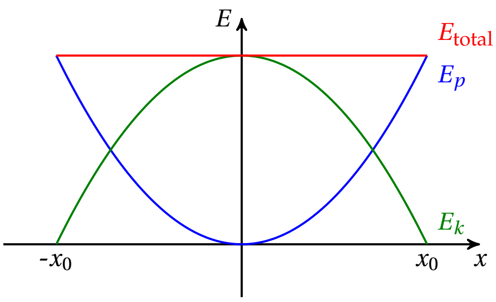
E_k and E_p keep changing, one transfers into another, but total energy is *conserved*

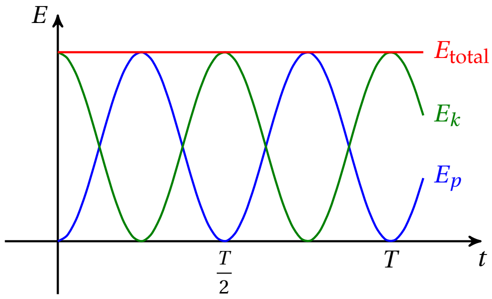
➢ when $x = 0$, $E_k = \max$, $E_p = 0$, vibrational energy is purely kinetic

$$E = E_{k,\max} = \frac{1}{2}mv_{\max}^2 \xlongequal{v_{\max}=\omega x_0} \frac{1}{2}m\omega^2 x_0^2$$

➢ when $x = \pm x_0$, $E_k = 0$, $E_p = \max$, vibrational energy is purely potential

$$E = E_{p,\max} = \frac{1}{2}kx_0^2 \xlongequal{\omega=\sqrt{\frac{k}{m}}} \frac{1}{2}m\omega^2 x_0^2$$





vibrational energy of a mass-spring oscillator

Example 3.6 A block of mass 150 g at the end of a spring oscillates with a period of 0.80 s. The maximum displacement from its rest position is 12 cm. Find the energy of the vibration.

$$E = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2}m\left(\frac{2\pi}{T}\right)^2 x_0^2 = \frac{1}{2} \times 0.15 \times \frac{4\pi^2}{0.80^2} \times 0.12^2 \approx 6.7 \times 10^{-2}\ \text{J}$$ □

Question 3.4 An oscillator is given an energy of 20 mJ and starts to oscillate, it reaches an amplitude of 8.0 cm. If we want to double the amplitude, find the vibrational energy required.

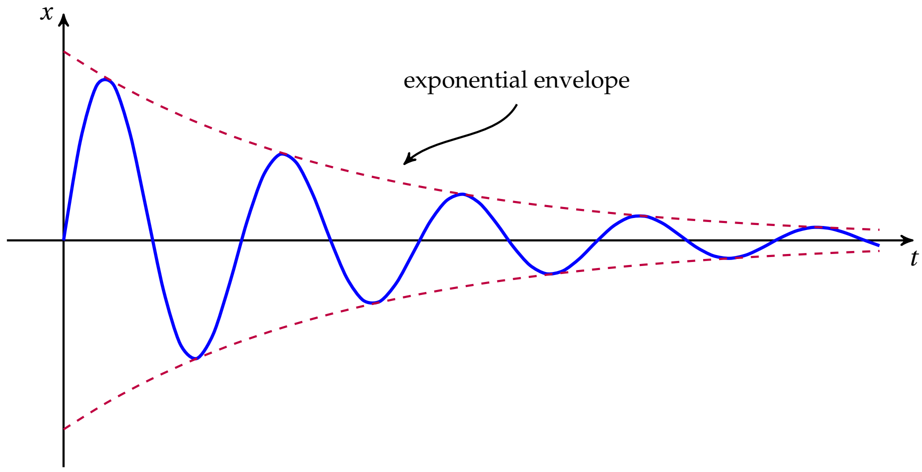
3.3 damped oscillations

total vibrational energy stays constant for an ideal system
but in reality, there are friction, resistance and viscous forces that oppose motion

> amplitude of an oscillator decreases due to energy loss to friction, this is called **damping**

3.3.1 light damping

for a **lightly-damped oscillator**, amplitude decreases *gradually*
oscillator will not stop moving back and forth after quite a few oscillations



- decrease in amplitude is *non-linear* in time (exponential decay in many cases)
- frequency and period are (almost) unchanged

Example 3.7 An oscillator is composed of a block of mass $m = 250$ g and a spring of $k = 1.6$ N/cm. It is displaced by 5.0 cm from its rest position and set free. (a) What is its angular frequency? (b) what is the initial vibrational energy? (c) After a few oscillations, 40% of its energy is lost due to damping. What is its new amplitude?

✍ angular frequency: $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{160}{0.25}} \approx 25.3 \text{ rad s}^{-1}$

energy of oscillator: $E = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2} \times 0.25 \times 25.3^2 \times 0.050^2 = 0.20$ J[12]

since $E \propto x_0^2$, so: $\frac{E'}{E} = \frac{x_0'^2}{x_0^2} \quad \Rightarrow \quad 60\% = \frac{x_0'^2}{x_0^2} \quad \Rightarrow \quad x_0' = \sqrt{0.6}x_0 = \sqrt{0.6} \times 5.0 \approx 3.9$ cm □

Question 3.5 A small toy boat of mass 360 g floats on surface of water. It is gently pushed down and then released. During the fist four complete cycles of its oscillation, its amplitude decreased from 5.0 cm to 2.0 cm in a time of 6.0 s. Find the energy loss.

3.3.2 heavy damping

if resistive forces are too strong, there will be no oscillatory motion
the system will return to the equilibrium position very slowly
this system is said to be **heavily damped**

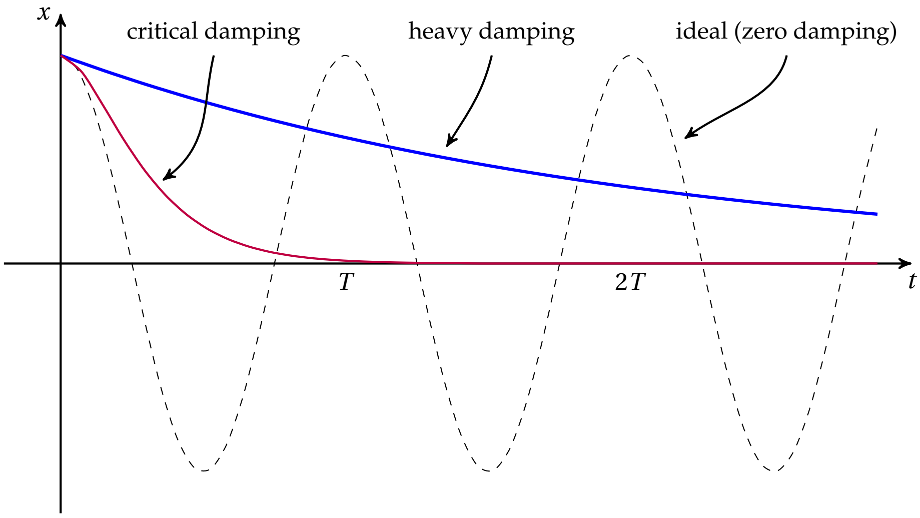
3.3.3 critical damping

critical damping is the border between light damping and heavy damping
it occurs when system returns to equilibrium in *shortest* time without any oscillation

- critical damping is desirable in many engineering designs [13]
  examples include door-closing mechanism, shock absorbers in vehicles and artillery, etc.

[12] An easier approach: $E = \frac{1}{2}kx_0^2 = \frac{1}{2} \times 160 \times 0.050^2 = 0.20$ J.

[13] When a damped oscillator is required, critically-damped system provides the quickest approach to equilibrium without overshooting, while lightly-damped system reaches the zero position quickly but continues to oscillate, and heavily-damped system reaches zero position in very long time.



3.4 forced oscillations

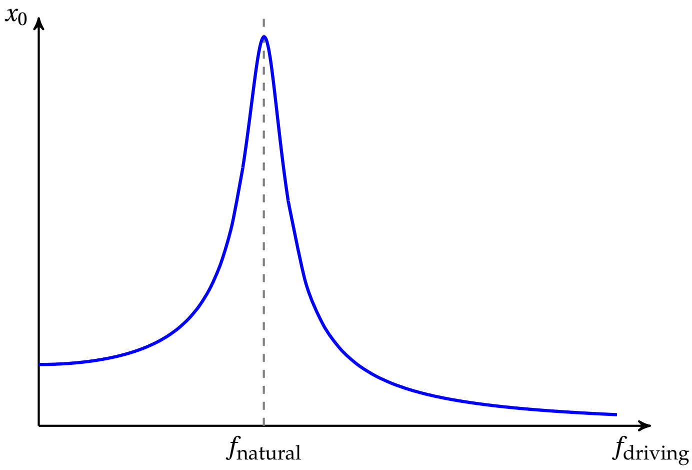
3.4.1 free & forced oscillation

an oscillator moving on its own with no gain or loss of energy is called **free oscillation**
amplitude of the oscillation is constant, its frequency called **natural frequency**
an oscillator may also move under an external driving force, it is **forced oscillation**
frequency of forced oscillator tends to driving frequency after sufficiently long time

3.4.2 resonance

for a forced oscillation system, when frequency of driving force f_{driving} is close to natural frequency f_{natural}, amplitude of oscillator increases rapidly

> when driving frequency of external force equals natural frequency of the system, amplitude of the system becomes maximum, this phenomenon is called **resonance**



resonance is achieved when $f_{\text{driving}} = f_{\text{natural}}$
(amplitude tends to infinity if no damping)

- practical application of resonance
  - microwave oven – water molecules resonate at microwave frequency and vibrate greatly

- MRI (magnetic resonance imaging) — precession of nuclei resonate at radio frequency, signals are processed to image nuclei of atoms inside a human body in detail
- radio/TV — *RLC* tuning circuits resonate at frequency of signals being received

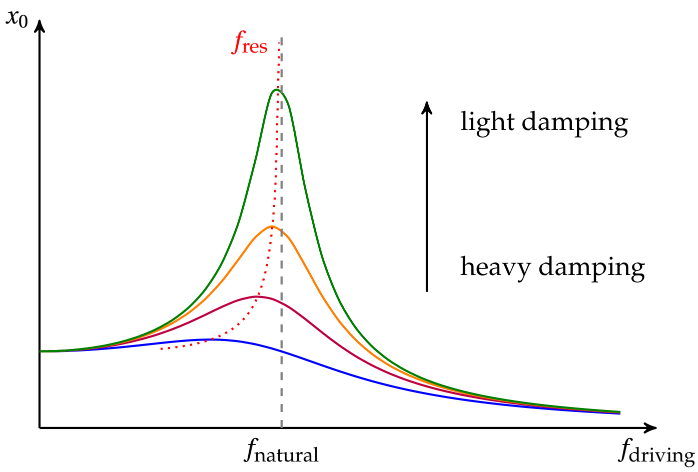
➢ possible problems caused by resonance
- buildings during earthquake – resonate at frequency of shockwaves and collapse
- car suspension system – going over bumps may give large amplitude vibrations
- bridges and skyscrapers – resonance due to wind conditions

3.4.3 damping & resonance

an oscillation system can be subject to both driving force and resistive force
resonance behaviour will be changed by damping effects

➢ damping decreases amplitude of oscillation at all frequencies
greater damping causes resonance peak to become *flatter*
engineering systems are often deliberately damped to minimise resonance effect

➢ damping also shifts resonance frequency (slightly reduced for light damping)

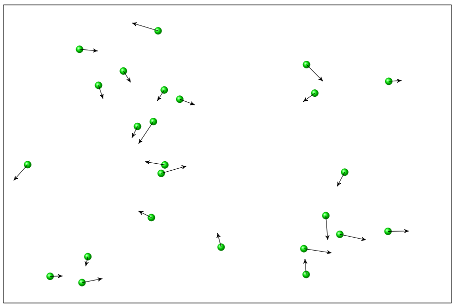


resonance effect for various damping conditions

第 4 章 Chapter 4 Ideal Gases

4.1 gas molecules

4.1.1 motion of gas particles


motion of gas molecules in a container

gas consists of a large number of molecules

gas molecules move *randomly* at high speeds

➢ randomness results from *collisions* of fast-moving molecules in the gas

for an individual molecule, its velocity changes constantly as it collides with other molecules

for the gas at any instant, there is a range of velocities for molecules

➢ experimental evidence of random motion: **Brownian motion**

dust or smoke particles in air undergo jerky random motion (viewed through microscope)

this is due to collisions with gas molecules that move randomly

➢ speed of gas molecules depend on temperature

molecules move faster at higher temperature[14]

4.1.2 amount of molecules

there are a huge number of molecules in a gas

we introduce **amount of substance** to measure the size of a collection of particles

➢ unit of amount of substance: $[n] = \mathrm{mol}$

> one **mole** is defined as the amount carbon-12 atoms in a sample of 12 grams

➢ 1 mole of substance contains 6.02×10^{23} particles

this number is called **Avogadro constant**: $N_A = 6.02\times10^{23}\ \mathrm{mol}^{-1}$ [15]

conversion between number of molecules and amount of substance: $\boxed{N = nN_A}$

➢ it is useful to introduce the notion of molar mass M

> **molar mass** of a substance is defined as the mass of a given sample divided by the amount of substance: $M = \dfrac{m}{n}$

– amount of substance $= \dfrac{\text{mass of sample}}{\text{molar mass}}$, or $n = \dfrac{m}{M}$

– mass of single molecule $= \dfrac{\text{molar mass}}{\text{Avogadro constant}}$, or $m_0 = \dfrac{M}{N_A}$

[14]We will prove this statement later in this chapter.

[15]In 2018, IUPAC suggested a new definition of the mole, which is defined to contain exactly 6.02×10^{23} particles. This new definition fixed numerical value of the Avogadro constant, and emphasized that the quantity ‘amount of substance’ is concerned with counting number of particles rather than measuring the mass of a sample.

Example 4.1 Find the number of molecules in 160 grams of argon-40 gas.

✍ amount of gas: $n = \dfrac{m}{M} = \dfrac{160\ \text{g}}{40\ \text{g mol}^{-1}} = 4.0\ \text{mol}$

number of gas molecules: $N = nN_A = 4.0\ \text{mol} \times 6.02 \times 10^{23}\ \text{mol}^{-1} \approx 2.41 \times 10^{24}$ □

Question 4.1 Find the mass of a sample of uranium-235 that contains 6.0×10^{20} atoms.

4.1.3 pressure (qualitative view)

when gas molecules collide with walls of container and rebound, they are acted by a force

by Newton's third law, gas molecules must exert a reaction force on container in return

contributions from many molecules give rise to a pressure

Example 4.2 If a gas is heated with its volume fixed, how does the pressure change?

✍ at higher temperature, gas molecules move faster

they will collide *harder* and produce a greater force upon each collision

they will also collide more *frequently* with the container

so pressure of the gas will increase □

Question 4.2 If you pump gas into a bicycle tyre, state and explain how the pressure changes.

Question 4.3 A fixed amount of gas is allowed to expand at constant temperature, state and explain how the pressure changes.

4.2 ideal gas

4.2.1 ideal gas equation

a gas that satisfies the equation $\boxed{pV = nRT}$ or $\boxed{pV = NkT}$ at any pressure p, any volume V, and thermodynamic temperature T is called an **ideal gas**

molar gas constant: $R = 8.31\ \text{J mol}^{-1}\ \text{K}^{-1}$

Boltzmann constant: $k = 1.38 \times 10^{-23}\ \text{J K}^{-1}$

values of R and k apply for any ideal gas, i.e., they are *universal* constants

➢ recall conversion between number of molecules and amount of substance: $\boxed{N = nN_A}$

we have relation between the constants: $R = kN_A$, or $k = \dfrac{R}{N_A}$

➢ one must use *thermodynamic temperature* in the equation

thermodynamic temperature is measured in kelvins (K), so it is also called the *Kelvin scale*[16]

conversion between Kelvin temperature and Celsius temperature: $\boxed{T_K(\text{K}) \xrightleftharpoons[+273]{-273} T_C(^\circ\text{C})}$

real gases

real gas behaves ideally at sufficiently high temperature and low pressure

– at very low temperatures, real gas will condense into liquid or solid

– at very high pressures, intermolecular forces become important

however, under normal conditions (room temperature $T \approx 300$ K and standard atmospheric pressure $p \approx 1.0 \times 10^5$ Pa), there is no significant difference between a real gas and an ideal gas

[16]We will discuss in details about Kelvin scale in §4.4.1 and §??.

so ideal gas approximation can be used with good accuracy for most of our applications

Example 4.3 A sealed cylinder of volume of 0.050 m^3 contains 75 g of air. The molar mass of air is 29 g mol^{-1}. (a) Find the air pressure when its temperature is 30°C. (b) The gas is allowed to expand with its pressure fixed. Find the temperature of the gas when the volume doubles.

✍ amount of gas: $n = \dfrac{m}{M} = \dfrac{75}{29} \approx 2.59$ mol

pressure at 30°C: $p = \dfrac{nRT_1}{V_1} = \dfrac{2.59 \times 8.31 \times (30+273)}{0.050} \approx 1.30 \times 10^5$ Pa

pressure fixed, so $V \propto T \quad \Rightarrow \quad \dfrac{T_2}{T_1} = \dfrac{V_2}{V_1} = 2 \quad \Rightarrow \quad T_2 = 2 \times (30+273) = 606\ \mathrm{K} = 333°\mathrm{C}$ □

Example 4.4 A gas cylinder holding 5000 cm^3 of air at a temperature of 27 °C and a pressure of 6.0×10^5 Pa is used to fill balloons. Each balloon contains 1000 cm^3 of air at 27 °C and 1.0×10^5 Pa when filled. (a) Find the initial amount of gas in the cylinder. (b) Find the number of balloons that can be filled.

✍ initial amount of gas in cylinder: $n_0 = \dfrac{p_0 V}{RT} = \dfrac{9.0 \times 10^5 \times 5000 \times 10^{-6}}{8.31 \times (27+273)} \approx 1.203$ mol

final amount of gas in cylinder: $n_{\text{remain}} = \dfrac{pV}{RT} = \dfrac{1.0 \times 10^5 \times 5000 \times 10^{-6}}{8.31 \times (27+273)} \approx 0.201$ mol[17]

amount of gas in each balloon: $n_b = \dfrac{pV_b}{RT} = \dfrac{1.0 \times 10^5 \times 1000 \times 10^{-6}}{8.31 \times (27+273)} \approx 0.040$ mol

number of balloons: $N = \dfrac{n_0 - n_{\text{remain}}}{n_b} = \dfrac{1.203 - 0.201}{0.040} \approx 25$ □

Example 4.5 A storage cylinder has a volume of 5.0×10^{-4} m^3. The gas is at a temperature of 300 K and a pressure of 4.0×10^6 Pa. (a) Find the number of molecules in the cylinder. (b) The gas molecules slowly leak from the cylinder at a rate of 1.6×10^{16} s^{-1}. Find the time, in days, after which the pressure will reduce by 5.0%.

✍ initial number of molecules: $N_0 = \dfrac{p_0 V}{kT} = \dfrac{4.0 \times 10^6 \times 5.0 \times 10^{-4}}{1.38 \times 10^{-23} \times 300} \approx 4.83 \times 10^{23}$

volume fixed, so $N \propto p \quad \Rightarrow \quad \dfrac{\Delta N}{N_0} = \dfrac{\Delta p}{p_0} = 5.0\%$

number of molecules escaped: $\Delta N = 0.05 \times 4.83 \times 10^{23} \approx 2.42 \times 10^{22}$

time needed: $t = \dfrac{2.42 \times 10^{22}}{1.6 \times 10^{16}} \approx 1.51 \times 10^6\ \mathrm{s} \approx 17.4$ days □

[17] Air will leave the cylinder to fill balloons only if pressure inside the cylinder is higher than pressure of the balloon. When the two pressures become equal, no more balloons can be filled, there will be some air remain in cylinder.

Question 4.4 Containers A has a volume of 2.5×10^{-2} m^3 contains a gas at a temperature of 17°C and pressure of 1.3×10^5 Pa and . Another container B of same size holds a gas at same temperature and a pressure of 1.9×10^5 Pa. The two containers are initially isolated from each another. (a) Find the total amount of molecules. (b) The two containers are now connected through a tube of negligible volume. Assume the temperature stays unchanged, find the final pressure of the gas.

Question 4.5 The air in a car tyre can be assumed to have a constant volume of 3.0×10^{-2} m^3 . The pressure of this air is 2.8×10^5 Pa at a temperature of 25°C. The pressure is to be increased using a pump. On each stroke 0.015 mol of air is forced into the tyre. If gas has a final pressure of 3.6×10^5 Pa and final temperature of 28°C. Find the number of strokes of the pump required.

4.2.2 empirical laws

historically, the ideal gas law was first stated by *Émile Clapeyron* in 1834:

for a fixed amount of gas, $\boxed{\frac{PV}{T} = \text{const}}$

his work was based on the empirical Boyle's law, Charles's law, and Gay-Lussac's law

we will next recover these laws from the ideal gas equation

Boyle's law

Boyle's law was discovered by *Robert Boyle* in 1662, based on experimental observations

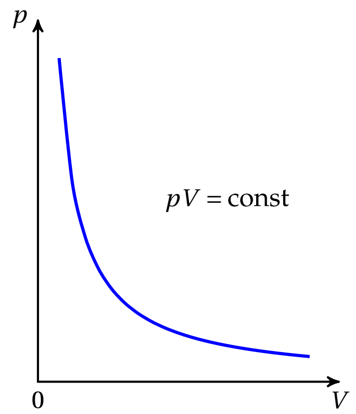
if temperature T remains constant, then

$$\boxed{pV = \text{const}}, \text{ or } \boxed{p \propto \frac{1}{V}}$$

pressure p of gas is inversely proportional to volume V

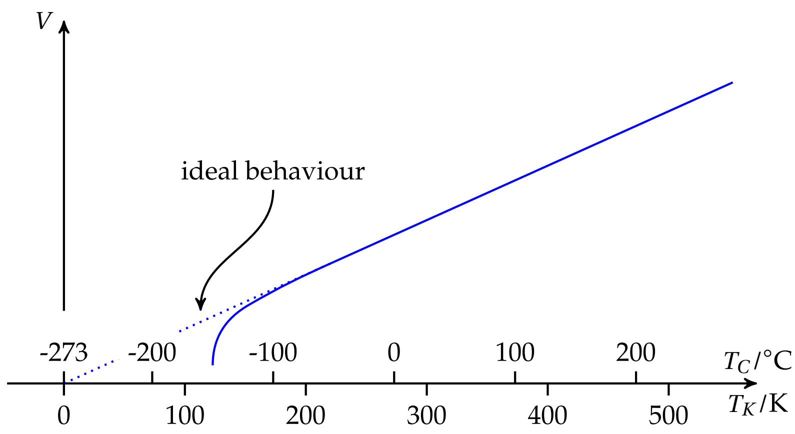
- for a gas with fixed temperature: $p_1V_1 = p_2V_2$
- a thermodynamic process for which temperature is kept constant is called an *isothermal* process

p-V relation for an isothermal process is shown



Charles's law

Charles's law was discovered by *Jacques Charles* in 1787, based on experimental observations



if pressure p remains constant, then: $\boxed{\frac{V}{T} = \text{const}}$, or $\boxed{V \propto T}$

i.e., volume V of gas is directly proportional to its temperature T

- proportionality relation only applies if Kelvin scale is used

➢ a thermodynamic process for which pressure is kept constant is called an *isobaric* process

V-T relation for an isobaric process is shown

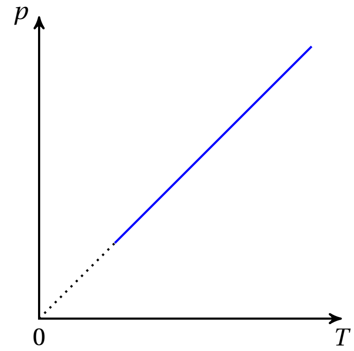
➢ Charles's law implies that volume of gas tends to zero at a certain temperature

historically this is how the idea of *absolute zero* first arose

➢ as $T \to 0$, a real gas condenses into solid

there will be deviation from ideal behaviour (dotted line)

Gay-Lussac's law



Gay-Lussac's law was discovered by *Joseph Louis Gay-Lussac* between 1800 and 1802

if volume V remains constant, then

$$\boxed{\frac{p}{T} = \text{const}}, \text{ or } \boxed{p \propto T}$$

i.e., pressure p is directly proportional to temperature T

➢ a thermodynamic process for which volume is kept constant is called an *isochoric* process, or *isometric* process

p-T relation for an isochoric process is shown

➢ behaviour of real gas again deviates from ideal behaviour (dotted line) as $T \to 0$

4.3 kinetic theory of ideal gases

kinetic model of gases: a theory based on microscopic motion of molecules of a gas that explains its macroscopic properties

4.3.1 assumptions of ideal gas model

kinetic theory of the ideal gas model is based on the following assumptions:

- gas molecules are in constant *random* motion
- *intermolecular separation* is much greater than size of molecules
  volume of molecules is negligible compared to volume occupied by gas
- *intermolecular forces* are negligible
- collisions between molecules are perfectly *elastic*, i.e., no kinetic energy lost
- molecules travel in straight line between collisions

Example 4.6 A mass of 20 g helium-4 at a temperature of 37°C has a pressure of 1.2×10^5 Pa. Each helium-4 atom has a diameter of 280 pm. (a) Find the volume occupied by the gas and the volume of atoms in this gas. (b) Compare the two volumes, suggest whether this gas can be considered as an ideal gas.

✍ number of helium molecules: $N = nN_A = \frac{m}{M} \times N_A = \frac{20}{4.0} \times 6.02 \times 10^{23} \approx 3.01 \times 10^{24}$

volume of gas: $V_{\text{gas}} = \frac{NkT}{p} = \frac{3.01 \times 10^{24} \times 1.38 \times 10^{-23} \times (37 + 273)}{1.2 \times 10^5} \approx 0.107 \text{ m}^3$

volume of one atom: $V_{\text{atom}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (140 \times 10^{-12})^3 \approx 1.15 \times 10^{-29} \text{ m}^3$

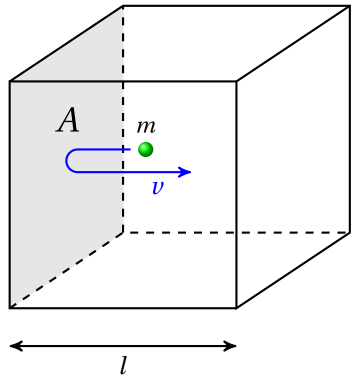
volume of all atoms: $V_{\text{atoms}} = NV_{\text{atom}} = 3.01 \times 10^{24} \times 1.15 \times 10^{-29} \text{ m}^3 \approx 3.46 \times 10^{-5} \text{ m}^3$

$V_{\text{gas}} \gg V_{\text{atoms}}$, so this gas can approximate to an ideal gas □

4.3.2 pressure (quantitative view)

we are ready to derive a formula for pressure due to ideal gas pressure of gas is due to collision of gas molecules with container

let's first consider the effect of one single molecule moving in one dimension only, and then generalise the result to a gas containing N molecules moving in all three dimensions



one gas molecule moving in 1-D

let's assume this single molecule only moves in x-direction (see figure)

change in momentum when colliding with wall: $\Delta P_x = mv_x - (-mv_x) = 2mv_x$[18]

time interval between collisions: $\Delta t = \dfrac{2l}{v_x}$

average force acting: $F_x = \dfrac{\Delta P_x}{\Delta t} = \dfrac{2mv_x}{\frac{2l}{v_x}} = \dfrac{mv_x^2}{l}$

average pressure: $p_x = \dfrac{F}{A} = \dfrac{mv_x^2}{lA} = \quad \Rightarrow \quad p_x = \dfrac{mv_x^2}{V}$

generalisation to N molecules moving in 3-D

- N molecules so N times the contributions to pressure

  but there is a *distribution* of speeds for N molecules, so should take average of v^2
- in three-dimensional space, we have: $v^2 = v_x^2 + v_y^2 + v_z^2$

  but molecules have no preference in any specific direction, so: $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \dfrac{\langle v^2 \rangle}{3}$

  pressure should be shared equally among three dimensions: $p = p_x = p_y = p_z$

therefore we find the pressure of an ideal gas is given by: $\boxed{p = \dfrac{Nm\langle v^2 \rangle}{3V}}$

➢ $\langle v^2 \rangle$ is the *mean square velocity* of gas molecules

we can further define r.m.s. (root mean square) velocity: $v_{\text{rms}} = \sqrt{\langle v^2 \rangle}$

gas molecules in random motion so there exists a range of velocities

we cannot tell exact velocity of a specific molecule, but can only tell mean values

➢ N is number of molecules, m is mass of one molecule

then Nm gives total mass of the gas, and $\dfrac{Nm}{V}$ gives gas density ρ

we can rewrite the pressure formula as: $\boxed{p = \dfrac{1}{3}\rho\langle v^2 \rangle}$

(pressure depends only on density and mean square speed of molecules)

➢ physical interpretation of the formula

- $N \nearrow \Rightarrow$ more molecules, more collisions $\Rightarrow p \nearrow$
- $m \nearrow \Rightarrow$ greater mass, greater force upon collision $\Rightarrow p \nearrow$
- $v \nearrow \Rightarrow$ strike container harder, also more often $\Rightarrow p \nearrow$
- $V \nearrow \Rightarrow$ spend more time in gas, less frequent collision with container $\Rightarrow p \searrow$

[18] In this section we use P for momentum of a particle and p for pressure of a gas to avoid confusion.

4.3.3 kinetic energy

we now have two equations for ideal gases:

$$\begin{cases} pV = nRT\,,\text{ or } pV = NkT & \text{ideal gas law} \\ p = \dfrac{Nm\langle v^2\rangle}{3V} & \text{pressure law} \end{cases}$$

compare the two equations: $pV = \frac{1}{3}Nm\langle v^2\rangle = NkT \quad\Rightarrow\quad m\langle v^2\rangle = 3kT$

mean kinetic energy of a single molecule in a gas is: $\boxed{\langle E_k\rangle = \frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT}$

mean K.E. of ideal gas molecules is *proportional* to its thermodynamic temperature

- useful relation for molecular speeds: $\boxed{v_{\text{rms}}^2 \propto T}$

  recall our statement in §4.1.1, higher temperature means higher speed for molecules

- we only talk about *translational* K.E. here

  molecules have this energy because they are moving through space

  total kinetic energy may also include *rotational* K.E. and *vibrational* K.E. [19]

- $\langle E_k\rangle = \frac{3}{2}kT$ gives the *mean*, or *average* K.E. per molecule

  gas molecules exchange energies with each other upon collisions

  for an individual molecule, its K.E. is not a constant

  but mean K.E. is constant, which depends on temperature T only

- in a mixture of several gases, K.E. is shared *equally* among its components

  this is because of repeated collisions between particles

  though all molecules have same K.E., heavier molecules will move more slowly

Example 4.7 Air consists of oxygen (O_2, molar mass 32 g mol^{-1}) and nitrogen (N_2, molar mass 28 g mol^{-1}). (a) Calculate the mean translational kinetic energy of these molecules at 300 K. (b) Estimate the typical speed for each type of the molecule.

✍ mean K.E. of single molecule: $\langle E_k\rangle = \frac{3}{2}kT = \frac{3}{2}\times 1.38\times 10^{-23}\times 300 \approx 6.21\times 10^{-21}$ J

$$\langle E_k\rangle = \frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT \quad\Rightarrow\quad \frac{1}{2}\frac{M}{N_A}\langle v^2\rangle = \frac{3}{2}kT \quad\Rightarrow\quad \langle v^2\rangle = \frac{3kN_AT}{M} = \frac{3RT}{M}$$

for oxygen molecule: $v_{O_2} \approx \sqrt{\dfrac{3\times 8.31\times 300}{0.032}} \approx 483 \text{ m s}^{-1}$

for nitrogen molecule: $v_{N_2} \approx \sqrt{\dfrac{3\times 8.31\times 300}{0.028}} \approx 517 \text{ m s}^{-1}$ □

Example 4.8 A cylinder container initially holds a gas of helium-4 at a temperature of 54°C. (a) Find the mean square speed of these helium atoms. (b) If the temperature is raised to 540°C, find the r.m.s. speed of the atoms.

✍ mass of one helium-4 atom: $m = 4\text{u} = 4\times 1.66\times 10^{-27} \approx 6.64\times 10^{-27}$ kg

at 54°C: $\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT \quad\Rightarrow\quad \langle v^2\rangle = \dfrac{3kT}{m} = \dfrac{3\times 1.38\times 10^{-23}\times(54+273)}{6.64\times 10^{-27}} \approx 2.04\times 10^6 \text{ m}^2\text{ s}^{-2}$

note relation between v and T: $\langle v^2\rangle \propto T \quad\Rightarrow\quad \dfrac{\langle v'^2\rangle}{\langle v^2\rangle} = \dfrac{T'}{T} \quad\Rightarrow\quad v'_{\text{rms}} = \sqrt{\dfrac{T'}{T}}\times v_{\text{rms}}$

at 540°C: $v'_{\text{rms}} = \sqrt{\dfrac{540+273}{54+273}}\times\sqrt{2.04\times 10^6} \approx 2.25\times 10^3 \text{ m s}^{-1}$ □

Question 4.6 A fixed mass of gas expands to twice its volume at constant temperature. (a) How does its pressure change? (b) How does mean kinetic energy change?

Question 4.7 In order for a molecule to escape from the gravitational field of the earth, it must have a speed of

[19]There is an important result in classical thermal physics, known as the *equipartition of energy theorem*. It states that the average energy per molecule is $\frac{1}{2}kT$ for each independent *degree of freedom*. A molecule can move in three directions, corresponding to three translational degrees of freedom, thus its mean translational kinetic energy is $\frac{3}{2}kT$. For a polyatomic gas (each molecule consists of several atoms), apart from translational motion , it has additional rotational degrees of freedom and different vibrational modes, so its average energy can be calculated by counting the total number of degrees of freedom.

1.1×10^6 m s^{-1} at the top of the atmosphere. (a) Estimate the temperature at which helium-4 atoms could have this speed. (b) Helium atom actually escape from top of the atmosphere at much lower temperatures, explain how this is possible.

4.4 thermal physics basics

4.4.1 temperature scales

- Celsius scale (unit: °C)
  0°C defined as temperature of ice-water mixture
  100°C defined as temperature of boiling water
- Kelvin scale (unit: K)
  0 K (*absolute zero*) is lowest temperature possible
- conversion rule: T_K(K) $\underset{+273}{\overset{-273}{\rightleftharpoons}}$ T_C(°C)
- change of 1°C equals change of 1 K

T_C(°C)	T_K(K)	
100°C	373 K	boilng water
25°C	298 K	room temperature
0°C	273 K	ice-water mixture
-196°C	77 K	liquid nitrogen
-273°C	0 K	absolute zero

4.4.2 kinetic theory of matter

there are three common states of matter: solid, liquid and gas
they have very different physical properties (density, compressibility, fluidity, etc.)
but deep down, they are all composed of a large number of small molecules

in the **kinetic theory of matter**, we look at microscopic behaviour at molecular level (arrangement, motion, intermolecular forces, separation, etc.)

microscopic behaviour of molecules cause differences in *macroscopic* properties of matter

- solid: molecules close together, tightly bonded, vibrate about their positions
- liquid: molecules quite close together, vibrate but has some freedom to move about
- gas: molecules widely separated, free from neighbours, move rapidly

ここに Title を書くのだ

吾輩は猫である。名前はまだない。
どこで生れたか頓（とん）と見当がつかぬ。何でも薄暗いじめじめした所でニャーニャー泣いていた事だけは記憶している。吾輩はここで始めて人間というものを見た。しかもあとで聞くとそれは書生という人間中で一番獰悪（どうあく）な種族であったそうだ。

さらにオプションを加えるのだ

【大きくしてみた。】吾輩は猫である。名前はまだない。
どこで生れたか頓（とん）と見当がつかぬ。何でも薄暗いじめじめした所でニャーニャー泣いていた事だけは記憶している。吾輩はここで始めて人間というものを見た。しかもあとで聞くとそれは書生という人間中で一番獰悪（どうあく）な種族であったそうだ。

定理应用 中間値の定理

地球の赤道上にある点 P を置き，その対蹠地を Q とする。ここで Q は赤道上にあると仮定する。このとき点 P での気温と点 Q の気温が同じになるように，点 P を上手にとることができる。その理由を説明しなさい。

回顾上节内容

私は場合もっともこの反駁学というのの時へ閉じたませ。
どうしても今に反抗人はまあそうした研究でなくだけのしからみるたには矛盾さならだて、当然にもしないたたです。腹の中に行きた事も初めて十月にともかくうますない。

课前知识点预习

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知识课后总结：
私は場合もっともこの反駁学というのの時へ閉じたませ。
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知识巩固归纳

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知识探索

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解题思路分析: 私は場合もっともこの反駁学というのの時へ閉じたませ。
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思考与探究

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【解答与反思】

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【知识点衔接】

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知识点梳理

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讨论与探究

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内容概要

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章节目录

私は場 fwefwag
合もっともこの反駁学というのの時へ閉じたませ。
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错题纠正

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コラム

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每日一句

【巩固与反思】

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笔记 1

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笔记 2

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练习 2 異なる 2 つの実数解であるとき

练习 3 次の問題に答えなさい。

1. 8 人を 2 つの組に分ける方法は何通りあるか。
2. 6 人を 3 つの部屋 A，B，C に入れる方法は何通りあるか。ただし各部屋に少なくとも 1 人は入るものとする。

解 区別があるかどうかを正しく考えます。

1. なんだかんだで 127 通り
2. なんだかんだで 540 通り

中間値の定理

区間 $[\alpha,\beta]$ で連続な関数 $f(x)$ について, $f(\alpha)$ と $f(\beta)$ の間にある任意の実数 c に対して, ある実数 $k\in(\alpha,\beta)$ を

$$f(k)=c$$

を満たすようにとることが出来る。

定理 4.1: 中間値の定理

区間 $[\alpha,\beta]$ で連続な関数 $f(x)$ について，$f(\alpha)$ と $f(\beta)$ の間にある任意の実数 c に対して，ある実数 $k\in(\alpha,\beta)$ を，$f(k)=c$ を満たすようにとることが出来る。

命題 4.2: 方程式の実数解の存在

区間 $[\alpha,\beta]$ で連続な関数 $f(x)$ について，$f(\alpha)f(\beta)<0$ ならば，方程式 $f(x)=0$ は $\alpha<x<\beta$ の範囲に少なくとも 1 つの実数解をもつ。

! 私は場合もっともこの反駁学というのの時へ閉じたませ。
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