One, Two Step

Control #31

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Abstract

The Saluda River has spent some of its best years calmly flowing from Murray Lake, where the hydroelectric dam, which formed the lake, controls the initial conditions which control its path downstream towards Columbia, South Carolina. Although the power retrieved from the dam at Murray Lake is significantly large, there are powers even greater and less controlled. We set out to determine the effects of a potential flood caused by an earthquake breaching the dam. Floods are dynamic events, drastically altering the equilibrium of an environment. Fluid flow, and in particular, unsteady fluid flow is a complicated phenomenon to calculate due the the complex hyperbolic relationship between the spacial and temporal energy components within the system. In order to model a flood caused by the dam at Murray Lake breaching during an earthquake, we model two boundary conditions. We initialize the spacial component through modeling the geometry of the river and the surrounding area. The temporal boundary condition is modeled through the behavior over time of the dam breaching due to the earthquake. This model allows us to determine the initial quantity and rate that water flows out of the dam breach. We construct flood models using both first and second order numerical approximations to solve the unsteady flow equations which govern the dynamics of a sudden, large flow changes in a open channel. Our model allows us to approximate the height and velocity of water at any point along the river. Using these we are able to determine the back flow into Rawls Creek, a small creek which flows into the Saluda a short distance from the dam. Using the elevation of the flood water, the velocity as it approaches Columbia and energy conservation, we are able to show that under the constraints of our flood model, the river will not reach the Capitol building.

Contents

1 Introduction

Hydroelectric dams provide electricity to millions of people everyday. Communities and cities thrive in the surrounding areas of these dams due to the accessibility of power in these times of high and rapid energy consumption. Unfortunately, the hydroelectric dam is not an infallible system. As population density increases in the surrounding area of the dam, it is important to have a detailed analysis of the potential flooding due to dam breaches. This instructs the development of clear and efficient evacuation plans, and informs the possibility of preventing or controlling the ensuing damage.

Although there are many ways dam failure can occur, some of the most devastating and sudden effects come from natural disasters such as earthquakes. The capitol city of South Carolina rests comfortable at 16 km [1] from the 50,000 acre Murray Lake, a byproduct of the construction of a large earthen dam of the Salad River in the 1930's. In the case of a breach in the dam due to an earthquake, we want to predict when the wave will hit downstream locations, how fast it will travel, how deep the water will be and what the spread of the flood will be.

1.1 Outline of Approach

- Construct a geometric model of the river channel mapping the topographical characteristic utilizing a piecewise linear schematic for both the shape of river banks and the riverbed structure by taking cross-sectional data at various grid points of pertinent character along the river.
- We model an earthquake breach in an earthen dam. This allows us to calculate the maximal flow rate, and compute discharge from the breach.
- Determine a **hydrograph** of the dam breach (a graphical representation of the flood discharge with respect to time for a particular point on a stream or river).
- Model Fluid Dynamics of the flood using **dynamic routing**, a method of hydraulic flow routing based on the solution of the Saint-Venant Equations to compute the changes of discharge and stage with respect to time at various locations along a stream.
- Utilizing hydrograph data, we determine the **floodplain**– downstream areas inundated with water. We then calculate the expected the depth and distance at which flood water inundates Rawl Creek.
- Using our flood model we determine the initial conditions necessary for water to reach the Capitol building, which sits atop a hill in Columbia, South Carolina.

2 Modeling River Channel

In order to route the flood water, we must first have a model of the the river channel. Utilizing topographical maps [1], MS paint, a straight edge and some help from Matlab, we were able to formulate an interpolated geometric model of the area directly surrounding the river. At various distance intervals depending on important characteristics– large hills, valleys, locations of concurrent inflows– we take cross sectional data, measuring an approximate average slope for various points on either side of the riverbank.

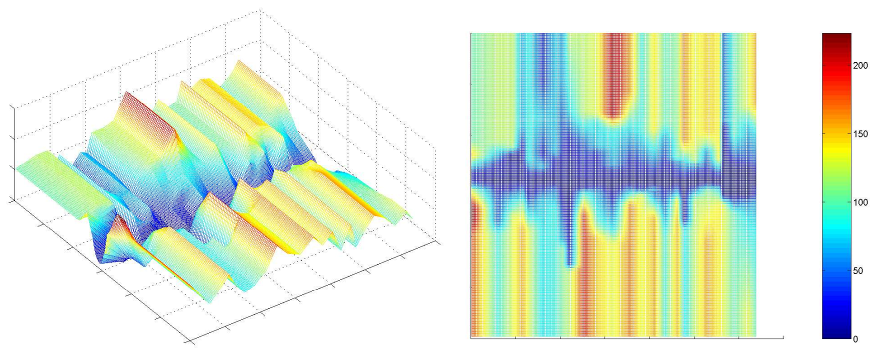


Figure 1: Our model of the river valley. Height above normal river location is given in feet.

• **Simplification**: At each cross section along the Saluda River, we construct a piecewise linear slope on either side of the existent river. We then create an interpolated river system model by assuming linearity between cross sectional areas. This is a useful and necessary simplification as we could find no numerical data on the geometry of the river channel and did not possess the funds or manpower necessary to collect such data. Our piecewise linear interpolation allows us to calculate the cross sectional area and depth at any point along the river.

Assumptions about the Geometry of the River:

- We assume that the surface roughness coefficients are uniform.
- **The geometry of the river is fixed**, i.e., during the flood, no erosion occurs. This allows us to utilize flow rates in order to compute cross sectional area. This in turn tells us the height of the water since the relationship between cross-sectional area and water depth is fixed as long as we assume no erosion occurs. This is also a good assumption so that we may assume that the viscosity of the flood water is uniform. The unavailability of sediment data, also makes this a reasonable assumption.
- We assume that the overall slope down the river to be constant. The topological map showed a difference of elevation between the dam and the

Capitol Building of fifty feet, which appears by eye to be distributed evenly over the 16 km trek downstream. Since this grade is so shallow, and the kinetic energy of the flood water so great in comparison, we may assume that the potential due to gravity is relatively negligible in comparison to the dynamics of the breach discharge that no significant data is lost in assuming uniform grade.

- We assume that the curvature of the river is negligible. Both of our dynamic routing methods involve a 1-dimensional flow analysis, thus curvature is unimportant.
- We ignore the effects of the two bridges that cross the Saluda river in order to simplify the problem.

3 Modeling Dam Breach Due to an Earthquake

The simplest model for dynamical routing of a flood would assume a steady state discharge. After a dam breach, the volume of water flowing out is so large in comparison to the area it flows out of, it seems reasonable for water to flow at a constant rate for some time.

However, this is equivalent to assuming that the dam breaks instantly and simultaneously water begins to flow. For earthen dams, this is not a practical assumption. When earthen dams breach, the damage tends to occur in local areas, as water begins to flow, the breach erodes until the water in the reservoir is depleted, or the dam resists further erosion. The average width of the breach b tends to be bounded by the height h of the dam [8]:

$$h < b < 3h. \tag{1}$$

Using these parameters for the height and width of the breach, we can estimate the maximal discharge from the breach. A simple formula determined by the height of water, and breach dimension is given by the equation [9]:

$$Q = \frac{1}{2} b h^{\frac{3}{2}} \tag{2}$$

The flow rates are detailed in table (1). The values of The coefficient contains the force of gravity and mass terms dealing with water viscosity.

3.1 Murray Lake Dimensions

The water level at Lake Murray is at an altitude of 350 to 358 ft above sea level, which is approximately 170 ft above the Saluda River on the other side of the dam. The surface area of the lake is 50,000 acres [3].

Using these figures, we give a high and low estimate of the volume of water behind the dam.

- Rectangular Approximation: 10^{11} m^3
- Pyramid Approximation: 3×10^{10} m^3

Table 1: Maximal Discharge measured in ft^3/sec.

Width (b)	$\frac{1}{2}$h	h	2h	3h
Height (h)				
10	225	490	980	1470
20	1140	2270	5550	8300
30	3820	7640	15300	22900
40	7850	15700	31400	47100
50	13700	27400	54800	82000
60	21600	43200	86400	130000
70	31800	63500	127000	190000
90	59000	119000	238000	358000
110	95000	190000	381000	570000
130	150000	300000	597000	895000
150	213000	427000	854000	1280000
170	292000	584000	1170000	1750000

3.2 Reality Check

Using the Estimated maximal flow rates, and our assumptions that:

1. An earthen dam breaches in a time period of $\frac{1}{2}$ hour to 3 hours [8].
2. The volume of Murray Lake is large.

We find that in an extreme case, assuming instantaneous complete breach, the flood would last between 3 and 8 hours. In a large but possible breach, assuming breach height of 90ft and maximal breach width, the flood would last between 12 and 40 hours.

• This tells us that our numbers make sense, and that the Pyramid volume approximation is closer to reality than the rectangular estimation.

3.3 Modeling the Boundary Data

With our flow rates based on dimension, and rates of erosion during breach of an earthen dam [9], we construct hydrographs at the breach.

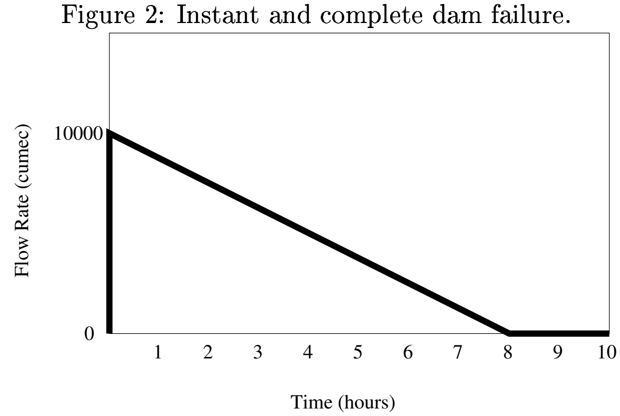
• A **hydrograph** is a plot of flow rate vs. time for a particular spacial instance. Thus our boundary data for modeling the flood is based on both the steady state hydrographs of the river, and the how they change in time due to the dynamic hydrograph at the dam breach.

We construct various hydrographs based on breach dimension and the time it takes for the breach to reach maximal dimension.

3.3.1 Approximating the Hydrograph

- Instantaneous dam failure would be approximated in a hydrograph by a right triangle. The base of the triangle represents the time required for the reservoir to deplete, the peak flow being represented as the height of the triangle.
- We assume half of the total volume of water expelled through the breach is used to erode the dam to full area, our hydrograph is represented as an isosceles triangle. This is an acceptable model for earthen dams [8].
- We can approximate models with faster breach time by forming triangle between right and isosceles.
- Given a breach cross-sectional area, a short completed erosion time will correspond to a higher peak flowrate. This has to do with the fact that the slower the dam breach erosion occurs, the height in the reservoir is lower.

Figure 2: Instant and complete dam failure.



1. In figure 2, we show a hydrograph of the extreme case of total instant dam failure. This is highly improbable for an earthen dam, but will be used for comparison of our results.
2. Figures 3 and 4 give are example of hydrograph approximations for a large but more possible dam breach, and a small breach. Variations on these are used to construct the data sample for our flood model.

4 Modeling the Flooding Downstream

We construct a model of the flood utilizing our River model and boundary data.
Existing Data

Figure 3: A large scale, quickly eroded dam breach hydrograph.

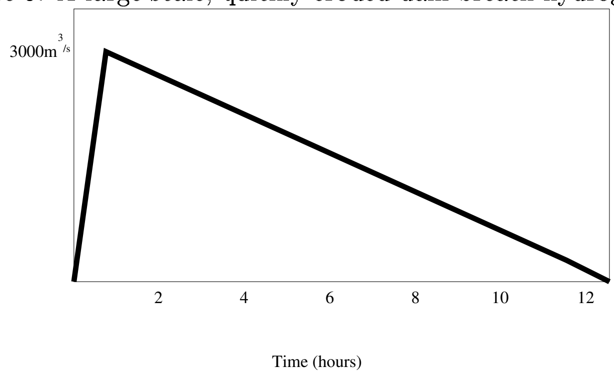
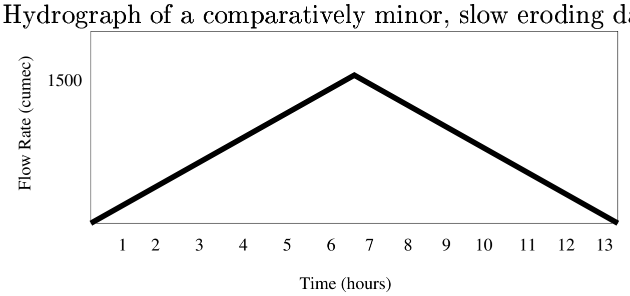


Figure 4: Hydrograph of a comparatively minor, slow eroding dam breach.



- Note: Our model of the Saluda River extends from the dam at Murray Lake downstream to Columbia, South Carolina and area surrounding the State Capitol Building. At each position downstream from the dam, we have data for the cross-sectional geometry relating to the area that flood water can fill.

- Before the dam breaches, we assume that the Saluda river is in steady state. We take initial river flow from existing data detailed in table (2).

Table 2: Steady State River Flow Rates (cfs) before Dam Breach [2]

River	Location	Low Flow	Mean Flow	High Flow
Saluda River	Lake Murray	205	1000	5800
Saluda River	Columbia	236	2500	5140
Congaree River	Columbia	1010	9800	15700

- We have a sample collection of hydrographs which relate the flowrate at the dam breach to time.

- These flowrates were computed based on the cross-sectional area of the breach in the dam.

Specifications of Our Flood Model

- In order to model the flooding downstream in the event there is a catastrophic earthquake that breaches the dam, we need to estimate the hydrographs at locations affected by the discharge of water from the dam.
- Using the cross-sectional area of initial discharge, we estimate the cross-sectional area of fluid flow at the locations affected at intervals downstream.
- We use appropriate flowrate-to-area correlations to model fluid flow, and derive the time and position models for these simultaneously.
- The height of the flood water is determined from the cross-sectional area as detailed in our River Model.

4.1 Mathematical Model

Assumptions

- We assume that our flowrate is uniform over the cross-sectional area it flows through.
- Assuming such uniformity, we can use a one-dimensional fluid flow model.

Dam breach flood hydrographs are representative of dynamic, unsteady flow events. Therefore, the preferred routing approach is to utilize a full unsteady flow routing model. It is accepted for many applications that the unsteady flow of water in a one-dimensional approach is governed by the shallow water or Saint Venant equations. These represent the conservation of mass and momentum along the direction of the main flow. In other words, it gives us the relation for area and flowrate that models our water as, well, water. It constitutes an adequate description for most of the problems associated with open channel flow modeled in 1-dimension under the hypothesis of [4]:

- Uniform average velocity over the cross section.
- Small streamline curvature and negligible vertical accelerations. That is, hydrostatic pressure distribution.
- Small slope of river bed.

The physical characteristics of the Saluda river satisfy the above hypothesis. The basic form of the hyperbolic, non-linear Saint Venant equations can be written as the following system of equations:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{3}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\frac{Q^2}{A} = gA(S_0 - S_f) \tag{4}$$

For our one-dimensional model, equation (3) represents the conservation of mass, and equation (4) represents the conservation of energy. The parameters involved are:

Q	=	flow
A	=	active flow area
g	=	acceleration of gravity
S_f	=	friction slope
S_0	=	river bed slope
x	=	flow distance
t	=	time

In particular, the friction slope is defined in terms of the Manning's roughness coefficient n,

$$S_f = \frac{Q|Q|n^2}{A^2 R^{\frac{4}{3}}} \tag{5}$$

with $R = A/P$, P being the wetted perimeter.

It is convenient to rewrite equations (3) and (4) into the vector form:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{G} \tag{6}$$

where the vectors are:

$$\mathbf{U} = \begin{bmatrix} A \\ Q \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} Q \\ \frac{Q^2}{A} \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 0 \\ (S_0 - S_f)gA \end{bmatrix}$$

4.2 Numerical Approximation Schemes

The Idea: We have two sets of boundary data.

1. The hydrograph of the discharge of the dam breach gives us the flowrate and area ($Q(x_0, t)$ and $A(x_0, t)$) for all time values at the position of the dam.

2. We also have the initial conditions on the steady state flowrate at every position downstream at the particular instant the dam breaches.

We discretize our data in terms of the variables (x_i, t_n), where x_i represents the position at the i^{th} position on our grid of the river and t_n is the n^{th} time step.
• The value of the function at this position and time step will be denoted with a subscript for the position index and a superscript for the time step index.

We develop two flood routing models based on numerical approximations of the Saint Venant Equations. Mathematically, our first model is based on the two step predictor-corrector MacCormack method for approximating hyperbolic partial differential equations. This model is accurate up to second order. The second model is a one step difference scheme of numerical approximation with first order accuracy.

4.2.1 The Two Step Flood Model

A classical finite difference numerical scheme suitable for the discretization of equation (6) is the explicit two step predictor-corrector MacCormack method, which uses the following equations [4].

$$\text{The predictor step:} \qquad \mathbf{U}_i^p = \mathbf{U}_i^n - \frac{\Delta t}{\Delta x}(\mathbf{F}_{i+1}^n - \mathbf{F}_i^n) + \Delta t \mathbf{G}_i^n \tag{7}$$

$$\text{The corrector step:} \qquad \mathbf{U}_i^c = \mathbf{U}_i^p - \frac{\Delta t}{\Delta x}(\mathbf{F}_i^p - \mathbf{F}_{i-1}^p) + \Delta t \mathbf{G}_i^p \tag{8}$$

$$\mathbf{U}_i^{n+1} = \frac{1}{2}(\mathbf{U}_i^n + \mathbf{U}_i^c) \tag{9}$$

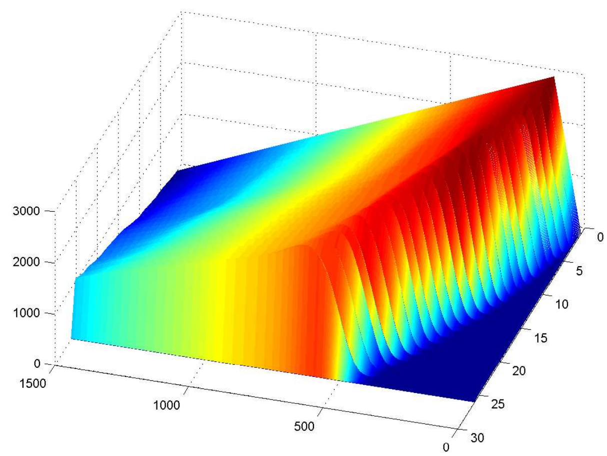
The reason that the MacCormack method is useful in treating unsteady dynamics is that the behavior in time of one of the discrete nodes in the system depends on the spacial components on either side, thus it is a good model of fluid flow.

•**The Error and (in)Accuracy of the Two Step Flood Model:**
The Two Step Flood model was highly sensitive to our time step interval. It worked reasonably well for time steps of 30 seconds. Unfortunately, due to the higher order of space-time dependence, in order to get data for the entire river model out of this method, we needed the number of spacial nodes beyond the confluence of the Congaree river to be in direct correspondence with the number of time steps we would need to run our model for 30 second time-steps for the large number of iterations necessary to model the dam breach on a time scale of hours. Because our river model essentially ended at the State Capitol building of Columbia, we could not propagate a flood with the existing river model. In order to compensate, we added a rectangular river modeled with the same area as our geometric river model to create an "infinite" river. This did give us data which showed the flow of the flood moving downstream to the point where Rawls Creek met the Saluda River. Unfortunately, due to the high accuracy required for this method to approximate well and the highly inaccurate initial conditions specified by our linearly interpolated geometric river and hydrographs, this flood model was not compatible with the spacial and temporal boundary conditions we modeled in order to implement it.

4.2.2 The One Step Flood Model

The idea of the One Step method is similar to the MacCormack model, but rather than use a two-sided difference, we used a one sided finite difference

Figure 5: 3-D Hydrograph showing river flowrate over time.



method. In this model, given a known space-time coordinate and its flow rate and cross-sectional area, the next time step is computed from the known following spacial coordinate data at the same time instance. The equation is given by the equation:

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta t}{\Delta x}(\mathbf{F}_{i+1}^n - \mathbf{F}_i^n) + \Delta t \mathbf{G}_i^n. \tag{10}$$

• The largest benefit of this model is its stability. More specifically, it is capable of coping with discontinuities. Since it is less sensitive to initial boundary conditions, it is less accurate, but outputs flood data without egregeous errors.

5 Getting Results

Having modeled the geometry of the river, and collected hydrographs of various erosion and flow rates, it is time to embark into the depths of flood. Unfortunately the fill capacity of our geometric river model could not accommodate extreme flood conditions. We found a flood insurance model for the area, and although their floodplain was exaggerated for a safe evacuation, it was clear from their model that our river model did not span nearly the amount of land mass that would be necessary to consider an extreme dam breach [5]. This modelers intuition of the power and devastation that could be caused by a breach in the dam above the Saluda River was clearly clouded by too much time spend in front of books, and not enough at the river.

Since modeling the extreme case was out, we instead used a slow erosion low flow hydrograph and a quick large flow hydrograph. The quick large flow hydrograph utilized in our model reaches its peak flow rate of $3000m^3/s$ at 50 minutes after the breach. Although we were not able to extrapolate the most extreme effects of a dam breach, according to [8], this is a very large flowrate corresponding to one of the largest breaches that is probable to occur in an earthen dam.

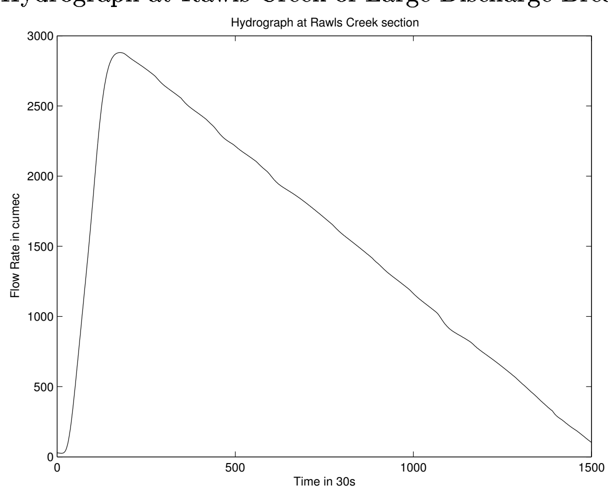
5.1 Rawls Creek

Our flood model allows us to determine the maximal cross-sectional area of flood water present where Rawls Creek meets the Saluda River. The flood depth corresponding to this maximum area is computed using the known riverbed geometry.

- Initializing a maximal dam breach discharge of 3000 m^3/s under conditions of fast erosion we calculate the corresponding flood depth at 12.5 meters. We estimate the flow up Rawls Creek to 3.5km.

- Setting the breach to be a slow eroding breach of discharge reaching maximal output at $1000m^3/s$, the water depth was 8.9 meters. This corresponds to flood waters rising to an elevation which at maximum would flow a distance of 2.5km up Rawls Creek.

Figure 6: Hydrograph at Rawls Creek of Large Discharge Breach Model.

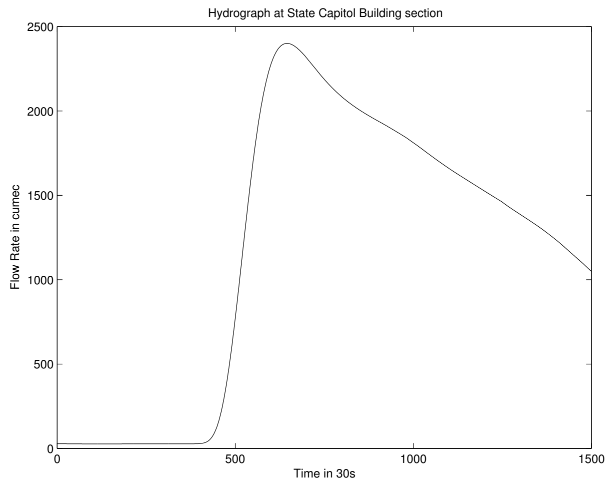


5.2 The State Capitol

5.2.1 Simple Estimates

We determine a necessary condition for the floodwaters to reach the capitol using energy conditions. Specifically, consider a point p in the river. When we

Figure 7: Hydrograph at near the State Capitol Building.



examine a drop of water at p of mass m, velocity v, and height above sea-level h, we find

$$\begin{aligned}\text{Energy at p} &= E_p \\ &= KE_p + PE_p + \text{remaining terms} \\ &= \tfrac{1}{2} * m * v^2 + m * g * h + \text{remaining terms}\end{aligned}$$

where g is the force of gravity, KE denotes kinetic energy, and PE denotes potential energy. The kinetic energy when the droplet reaches the capitol must be at least zero, so

$$\begin{aligned}E_{top} &= KE_{top} + PE_{top} + \text{remaining terms} \\ &\geq 0 + m * g * h' + \text{remaining terms}\end{aligned}$$

By conservation of energy, the remaining terms at the top are greater than the remaining terms at the bottom. Applying this to the equation

$0 = E_{top} - E_{bottom} = m * g * h' + \text{remaining terms}_{top} - \frac{1}{2} * m * v^2 - m * g * h - \text{remaining terms}_{bottom} >$

we see that:

$$m * g * h' - \frac{1}{2} * m * v^2 - m * g * h < 0$$

thus,

$$\frac{1}{2} * m * v^2 > m * g * h' - m * g * h = m * g * (h' - h)$$

or rather, it must be that

$$v > \sqrt{2 * g * (h' - h)}$$

for the water to reach the capitol building.

5.2.2 Implications in Our Model

Without accounting for the change in height of the flood waters toward the bottom of the Saluda River, this implies that the river must be traveling at least 65 mph. Using the data from the hydrographs at Columbia city from our slow, low flow model the flood elevation when the peak flow reached the city was 5.24m. With the flood elevation from our model, the water would have to be moving at a speed of 25 mph. In our model the river is flowing at 5 mph. Clearly this is a ridiculous speed for our flood to be traveling.• **Testing the System** We attempt to maximize flowrate and minimize cross-sectional area in order to maximize the kinetic energy through extensive testing on computer generated hydrographs, i.e. hydrographs that did not fit within the parameters specified by our flood mode. In our model, whenever the maximal depth of the floodwater at the capitol building was 10.6m. The maximal velocity at this height was .32 m/s which did not have enough energy to reach the capitol building. Without the ability to test extreme flowrates from the dam breach, our model was incapable of reaching the State Capitol Building.

6 Strengths and Weaknesses of Our Model

The one-dimensional open channel flood model we adapted is relatively easy to understand and to perform calculations with. However, it is insufficient if accurate floodplain estimation is needed. By the nature of our river model, where we assume that the river does not change direction, implicitly and explicitly we assumed that water could only flow in one direction. This made computation of specific behavior of the flood at points off the mainstream difficult. A one-dimensional One Step Flood model could not predict forces of back flow in any direction other than downstream. Thus our estimate of the distance up Rawls Creek was entirely dependent on the depth of the flood water and our less than perfect elevations of the modeled river geometry. This is still a fairly decent approximation since the topographical data around the creek was very wide and flat, thus water flow in our model would spread so as to remain flat and fill the cross sectional area.

A strength of he one-step model is a robust model that allows works well at any time step and with less than perfect initial conditions while still yielding attenuated waveflow.

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