Team Control Number

For office use only

T1 ________________

T2 ________________

T3 ________________

T4 ________________

22063

For office use only

F1 ________________

F2 ________________

F3 ________________

F4 ________________

Problem Chosen

A

2013 Mathematical Contest in Modeling (MCM) Summary Sheet

The Ultimate Brownie Pan

Abstract

Brownie Cake is famous for its fantastic taste, but how can we cook Brownie Cake better? Apparently, it needs to be baked in a suitable pan. Now, let's take a close look at Ultimate Brownie Pan designed in this article.

For the sake of dealing with the contradiction between delicacy and efficiency, there are two mathematical models being developed.

The model 1, namely, model with three-dimensional differential equation of heat conduction, provides a solution to studying the heat distribution of pans in different shapes. By using ANSYS to simulate, the maximum temperature difference of rectangle pan is 2.296 centigrade. And the maximum temperature difference of round pan is 1.386 centigrade. It proved that the heat temperature distribution of round pan is more uniformer than rectangle pan.

In order to study the best shape of pan under given conditions, model 2 is a mixed one of two-dimensional packing model and combination optimization model. Temperature variance of pan and utilization radio of area are selected to quantize two indexes given.Six basic shapes of pans are discussed.We get the best appropriate pan in the different shapes. In particular, a curve have been fitted to reflect which shape of pan is suitable for the oven under certain condition.

Keywords: Brownie Pan Heat Conduction Equation ANSYS Temperature Variance Combinatorial Optimization Model

Contents

I. Introduction

Product gets overcooked at the corners when baked in a rectangular pan, but not overcooked at the edges when baked in a round pan. However, using round pan is not efficient with respect to using space in a rectangular oven. So it is necessary to design pans of new shape satisfied for maximizing both even distribution of heat for the pan and number of pans that can fit in the oven.

In order to study better the heat distribution of pans in different shape, the paper develops a mathematical model with three-dimensional differential equation of heat conduction, and use ANSYS to simulate.

For the sake of studying the best shape of pans, the model 2 is developed as a mixed one of two-dimensional packing model and combination optimization model.

II. Models

1.Model 1

1.1 Symbols and Definitions

Table.1 Symbols and Definitions

symbol	definition
$T(x,y,z,t)$	temperature of a plate（$°C$）
$g(x,y,t)$	volumetric energy of heat source (Wm^{-3})
$\bar{x}(t),\bar{y}(t)$	functions describing the movement of the heat source（m）
T_0,T_1	temperatures of a surrounding medium（$°C$）
a,b,h	dimensions of a plate（m）
t	time（s）
x_0,y_0	coordinates of fixed point of a plate（m）
$G(x,y,z,t,\xi,\eta,\zeta,\tau)$	Green’s function
κ	thermal diffusivity（m^2s^{-1}）
ε	size of the quadratic element on the plate

	surface heated by the heat source（m）
$\delta(\,)$	Dirac delta function
α_0,α_1	heat transfer coefficients（$Wm^{-1}K^{-1}$）
λ	thermal conductivity ($Wm^{-1}K^{-1}$)
φ	angular velocity of the moving heat source ($rad\,s^{-1}$)
i,j,k,l,m,n	indices

1.2 Assumptions

For the model 1, some reasonable assumptions are as follows:

1.In the oven, the heat transfer mode is heat conduction;

2.The heat distributes evenly in the oven, and thermal environment of the same pans has no difference;

3.The pan's material is even and appears isotropy in physics.

1.3 The foundation of model 1

The temperature $T(x,y,z,t)$ of the pan satisfies differential equation of heat conduction:

$$\nabla_3^2 T - \frac{1}{k}\cdot\frac{\partial T}{\partial t} + \frac{1}{\lambda} g(x,y,t) = 0 \tag{1}$$

where $\nabla_3^2 \equiv (\partial^2/\partial x^2)+(\partial^2/\partial y^2)+(\partial^2/\partial z^2)$, k is thermal diffusivity, λ is thermal conductivity and $g(x,y,t)$ denotes a volumetric energy generation. [1]In consideration that the temperature rises rapidly to a stable state in the oven, the volumetric energy of heat source $g(x,y,t)$ is thought to be a constant.

The analysis for the heat distribution of the rectangular pan as a example is presented, as follows.

The differential equation (1) is complemented by the following initial and boundary conditions:

$$T(x,y,z,0) = 20 \tag{2}$$

$$\lambda\,\partial T(0,y,z,t)/\partial x = -\alpha_0\left[T_0 - T(0,y,z,t)\right] \tag{3}$$

$$\lambda\,\partial T(a,y,z,t)/\partial x=\alpha_1\left[T_1-T(a,y,z,t)\right] \tag{4}$$

$$\lambda\,\partial T(x,0,z,t)/\partial y=-\alpha_2\left[T_2-T(x,0,z,t)\right] \tag{5}$$

$$\lambda\,\partial T(x,b,z,t)/\partial y=\alpha_3\left[T_3-T(x,b,z,t)\right] \tag{6}$$

$$\lambda\,\partial T(x,y,0,t)/\partial z=-\alpha_4\left[T_4-T(x,y,0,t)\right] \tag{7}$$

$$\lambda\,\partial T(x,y,h,t)/\partial z=\alpha_5\left[T_5-T(x,y,h,t)\right] \tag{8}$$

where $\alpha_i\left(i=0,1,2,\cdots 5\right)$ is the heat transfer coefficients, and $T_i\left(i=0,1,2,\cdots 5\right)$ is known temperatures of the surrounding medium.

1.4 Solution to the problem

The solution to the initial-boundary problem, which is given by Eq.(1) and conditions (2)—(8), is determined by using Green's function method. The Green's function $G(x,y,z,t,\xi,\eta,\zeta,\tau)$ is a solution to the differential equation:

$$\left(\nabla_3^2-\frac{1}{k}\cdot\frac{\partial}{\partial t}\right)G(x,y,z,t,\xi,\eta,\zeta,\tau)=0 \tag{9}$$

and satisfies the homogeneous initial-boundary conditions, analogous to the initial and boundary conditions (2)—(8):

$$\left(\lambda\,\partial G/\partial x-\alpha_0 G\right)\Big|_{x=0}=0,\left(\lambda\,\partial G/\partial x+\alpha_1 G\right)\Big|_{x=a}=0 \tag{10}$$

$$\left(\lambda\,\partial G/\partial y-\alpha_2 G\right)\Big|_{y=0}=0,\left(\lambda\,\partial G/\partial y+\alpha_3 G\right)\Big|_{y=b}=0 \tag{11}$$

$$\left(\lambda\,\partial G/\partial z-\alpha_4 G\right)\Big|_{z=0}=0,\left(\lambda\,\partial G/\partial z+\alpha_5 G\right)\Big|_{z=h}=0 \tag{12}$$

The Green's function for the considered heat conduction problem as a solution to the homogeneous differential problem (9)—(12) is presented[2]. The application of a reciprocity relation [3]: $G(x,y,z,t,\xi,\eta,\zeta,\tau)=G(\xi,\eta,\zeta,-\tau,x,y,z,-t)$, in Eq.(9), yields

$$\left(\overline{\nabla}_3^2-\frac{1}{k}\cdot\frac{\partial T}{\partial\tau}\right)G(x,y,z,t,\xi,\eta,\zeta,\tau)=0 \tag{13}$$

where $\overline{\nabla}_3^2=\left(\partial^2/\partial\xi^2\right)+\left(\partial^2/\partial\eta^2\right)+\left(\partial^2/\partial\zeta^2\right)$. Eq.(13) is then used to solve the problem.

The Green's function is applied to determine the temperature T in the pan. To this end the following steps should be performed:

—Replacement of variables x, y, z, t in Eq.(1) by ξ, η, ζ, τ, respectively.

—Multiplication of both sides of the equation obtained in the first step, by the Green's function $G(x, y, z, t, \xi, \eta, \zeta, \tau)$.

— Integration of both sides of the equation obtained in the second step, with respect to ξ, η, ζ, τ in the intervals $(0, a), (0, b), (0, h), (0, t)$,respectively[1].

As a result one obtains

$$\int_0^a \int_0^b \int_0^h \int_0^t \left[\left(\overline{\nabla}_3^2 - \frac{1}{k} \cdot \frac{\partial T}{\partial \tau} \right) T(\xi, \eta, \zeta, \tau) + \frac{1}{\lambda} g(\xi, \eta, \zeta, \tau) \right]$$

$$\times G(x, y, z, t, \xi, \eta, \zeta, \tau) \, d\tau \, d\zeta \, d\eta \, d\xi = 0 \tag{14}$$

Next, the integral in Eq.(14) is integrated by parts: the terms which include derivatives of the function T with respect to ξ, η, ζ, are integrated by parts twice and the term including the derivative with respect to τ is integrated once. After utilizing the initial and boundary conditions (2)—(8) and Eq.(10)—(12), the following equation is obtained:

$$\int_0^a \int_0^b \int_0^h \int_0^t \left\{ \left[\left(\overline{\nabla}_3^2 - \frac{1}{k} \cdot \frac{\partial T}{\partial \tau} \right) G(x, y, z, t, \xi, \eta, \zeta, \tau) \right] \right.$$

$$\left. \times T(\xi, \eta, \zeta, \tau) + \frac{1}{\lambda} g(\xi, \eta, \zeta, \tau) \, G(x, y, z, t, \xi, \eta, \zeta, \tau) \right\}$$

$$\times d\tau \, d\zeta \, d\eta \xi + B.c. = 0 \tag{15}$$

where $B.c. = \int_0^b \int_0^h \int_0^t \left[\mu_1 T_1 G\big|_{\xi=a} + \mu_0 T_0 G\big|_{\xi=0} \right] d\tau \, d\zeta \, d\eta$

$$+ \int_0^a \int_0^h \int_0^t \left[\mu_3 T_3 G\big|_{\eta=b} + \mu_2 T_2 G\big|_{\eta=0} \right] d\tau \, d\zeta \, d\xi$$

$$+ \int_0^a \int_0^b \int_0^t \left[\mu_5 T_5 G\big|_{\zeta=h} + \mu_4 T_4 G\big|_{\zeta=0} \right] d\tau \, d\eta \, d\xi \tag{16}$$

Finally, using (13) in Eq.(15) and using the properties of the Dirac delta function[1], one obtains:

$$T(x, y, z, t) = \frac{1}{\lambda} \int_0^a \int_0^b \int_0^h \int_0^t g(\xi, \eta, \zeta, \tau) \, G(x, y, z, t, \xi, \eta, \zeta, \tau) \times d\tau \, d\zeta \, d\eta \, d\xi + B.c. \tag{17}$$

Analogous to the analysis method for the heat distribution of the rectangular pan in space rectangular coordinate system, problem of the heat distribution in a round pan based on cylindrical coordinate system is solved , and also the other cases such as elliptical, regular hexagon and so on.

1.5 Verification with simulation diagram

In order to show the distribution of heat across the outer edge of a pan for pans of different shapes - rectangular to circular and other shapes in between, six pans made up of iron in different shapes are selected to do simulation by using ANSYS[4]. Here are the physical properties of iron, requirements for baking[5] and six basic shapes of pans:

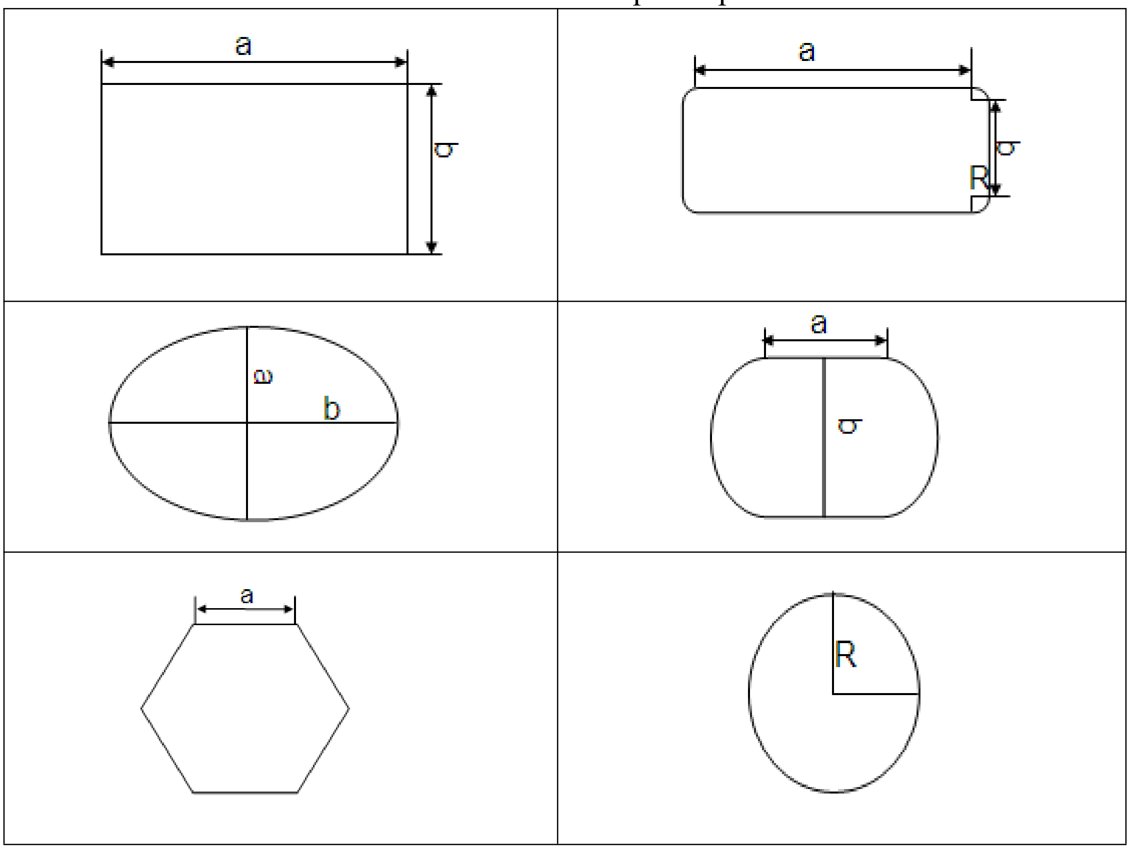
Table.2 The physical properties of iron

Items	Specific Heat	Density	Thermal Conductivity
Value	$460 J\cdot kg^{-1}\cdot °C^{-1}$	$7850 kg\cdot m^{-3}$	$40\times1.163 W\cdot m^{-1}\cdot °C^{-1}$

Table.3 Requirements for baking

Items	Baking Temperature	Size of Cake	Baking Time
Value	180°C	9cm×12cm	1800s

Table.4 Six basic shapes of pans



Simulation diagrams are as follows:

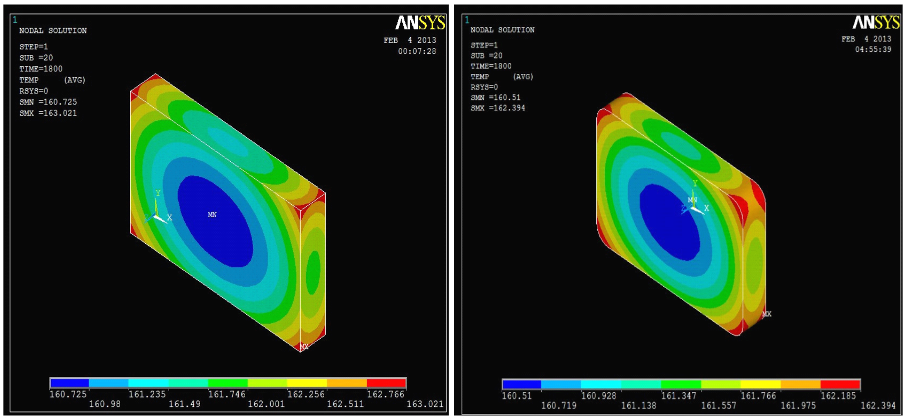


Figure.1 Temperature distribution simulation diagram one

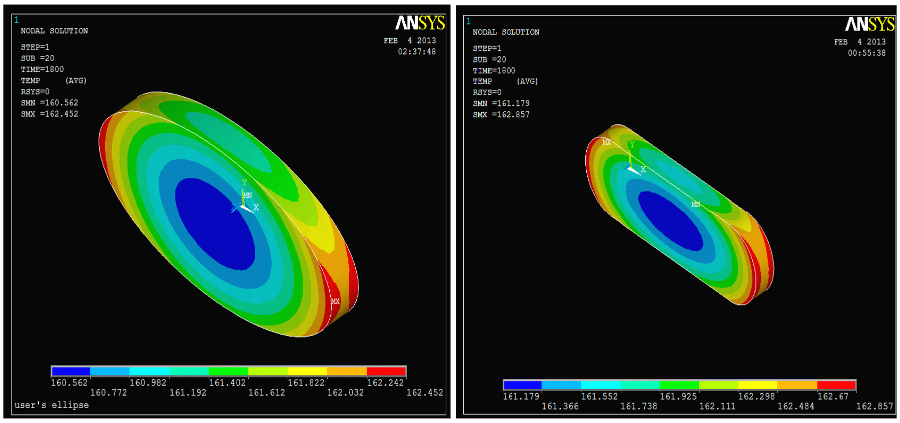


Figure.2 Temperature distribution simulation diagram two

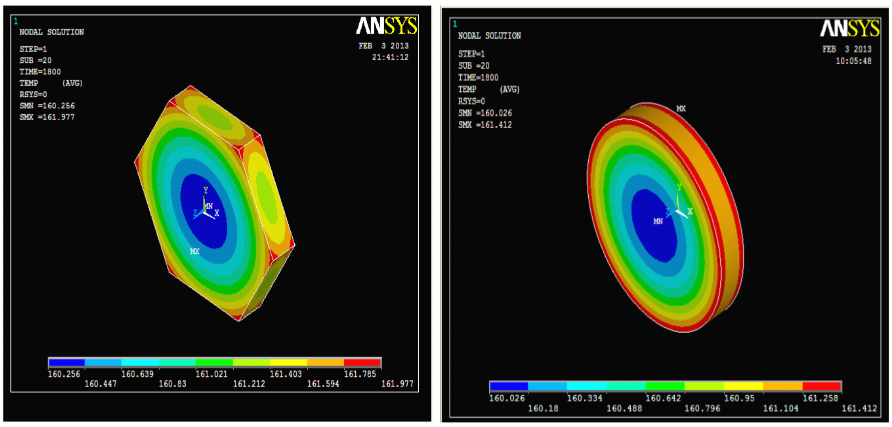


Figure.3 Temperature distribution simulation diagram three

Fig.1—3 show that the heat distribution of the pan becomes more and more uniform, with the pan's shape changing from rectangular to circular gradually. Therefore, it expounds perfectly that product gets overcooked at the corners when baked in a rectangular pan, but not overcooked at the edges when baked in a round pan.

1.6 Strength and Weakness

The advantages:

1.The model can be simulated by utilizing ANSYS ,which turns out the model is right and suitable.

2.The model is universal and can easily slove out heat distribution for different shapes of pans.

3.The model can appear the change of heat distribution for different shapes from rectangle to circular well.

The disadvantage:

The model considers heat conduction and heat convection but ignores heat radiation ,which leads to some errors to the result.

1.7 Conclusion:

Utilizing ANSYS to simulate the model 1,the paper gets Fig.(1)—(3) simulation diagrams for different geometric shapes, Fig.(1)—(3) indicate that when the shape changes from rectangle to circular,the temperature distribution for geometric shape becomes more and more uniform,so the temperature distribution for a round pan is most uniform.

2.Model 2

2.1 Symbols and Definitions

Table.5 Symbols and Definitions

symbols	definitions
u	the even extent of heat distribution in a pan
v	the area of pans to the area of oven inside ratio
p	the weight for the even extent of heat distribution in a pan
$1-p$	the weight for area ratio v
S	the area of oven inside
W	the width of the oven inside
L	the length of the oven inside
A	the area of a rectangular pan

a,b	the dimensions of the rectangular pan
n	the number of the rectangular pans
d_0	the temperature variance of round pan's outer edge
d'	the temperature variance of rectangular pan's outer edge

2.2 Assumptions

For the model 2, some reasonable assumptions are as follows:

1.The heat distributes evenly in the oven, and thermal environment of the same pans has no difference;

2.At the same time there is only one kind of shape of pans in the oven.

2.3 The foundation of model 2

The problem expounded is the two dimensional packing of a big rectangle with a given number of small rectangles or some other shapes. The analysis for the configuration of the rectangular pan in the oven as a example is presented, as follows.

It is important to consider that the sides of small rectangles must be parallel with the sides of a big rectangle. Two cases are possible: in the first case, a 90° degree rotation for smaller rectangles is not allowed, while the second case allows rotation. Smaller rectangles must be placed completely inside the big rectangle and they must not overlap. Let the dimensions of the big rectangle be respectively W and L. There are n small rectangles. Let their dimensions be respectively a_i and b_i $(i=1,2,\cdots n)$. It is allowed for the small rectangles to have the same dimensions.

Let x_i and y_i $(i=1,2,\cdots,n)$ be coordinates of the centers of rectangles in the rectangular coordinate system, where strip starts from x-axis and is spreading in the direction of y-axis. Bounding rectangle then has vertices in points with coordinates $(0,0)$, $(0,L)$, (W,L) and $(W,0)$. Model is presented and proofed in [24]. The following mathematical model for packing rectangles into bounding rectangle is obtained[6]:

$$\max\sum_{i=1}^{n}\frac{\max(0,L-y_i)}{L-y_i}a_ib_i \tag{18}$$

$$
s.t.\begin{cases}
\left|\dfrac{x_j-x_i}{a_i+a_j}-\dfrac{y_j-y_i}{b_i+b_j}\right|+\left|\dfrac{x_j-x_i}{a_i+a_j}+\dfrac{y_j-y_i}{b_i+b_j}\right|\ge 1,\quad i=1,2,\cdots n-1,\quad j=i+1,\cdots n \\
\left|y_i-L\right|\ge \dfrac{b_i}{2},\quad i=1,2,\cdots n \\
\dfrac{a_i}{2}\le x_i\le W-\dfrac{a_i}{2},\quad i=1,2,\cdots n \\
y_i\ge \dfrac{b_i}{2},\quad i=1,2,\cdots n
\end{cases} \tag{19}
$$

In order to determine the even extent of heat distributing in the rectangular pan exactly, it is necessary to separate the outer edge of the rectangular pan to several sections evenly. Then discretize the temperatures of each section, and obtain their variance. So the temperature variance of pan's outer edge could be satisfied for describing the even extent of heat distribution. The lesser the variance is, the better the even extent will be. As a result one obtains:

$$
\max Z = p\cdot v+\left(1-p\right)\cdot u \tag{20}
$$

$$
s.t.\begin{cases}
S=WL \\
A=ab \\
nA\le S \\
u=d_0/d' \\
v=nA/S
\end{cases} \tag{21}
$$

where u: the even extent of heat distribution in a pan;

v: the area of pans to the area of oven inside ratio;

p: the weight for area ratio v;

$1-p$: the weight for the even extent of heat distribution in a pan;

S: the area of oven inside;

W: the width of the oven inside;

L: the length of the oven inside;

A: the area of a rectangular pan;

a,b: the dimensions of the rectangular pan;

n: the number of the rectangular pans;

d_0: the temperature variance of round pan's outer edge;

d': the temperature variance of rectangular pan's outer edge;

2.4 Solution to the problem

BL algorithm must meet BL condition,that is, arbitrary rectangle within the configuration diagram can't be moved to left or down unless it produces interference or breaks the boundary.

Define the rectangle which will be arranged as P1,P2,P3.……Pn.

The algorithm has three steps as follows:

1.P1 is arranged in the bottom left corner,if the value of P1 is negative number,then rotating P1 for 90 degree,and working out the maximum height of the rectangular area parked after arranging.

2.Pi (i=2,3.……n) is arranged in the maximum height on the right side and moved to left or down in turn. First, Pi is moved to down as much as possible,then turned left as much as possible until the movement stops,finally,working out the the maximum height.

3.Repeating to do listed above until all rectangles are arranged,the maximum height got in the end is the length of the space which is parked[7].

In order to optimize a combination of conditions (1) and (2) where weights p and (1- p) are assigned to illustrate how the results vary with different values of W/Land p. As it known all, it is rather difficult to take W/L and p at the same time. In this paper, the change of W/L and p were considered separately.

Firstly, only the variation of p is taken into account, the change of W/L is not considered. Under this circumstance, different shapes of pans were placed in the oven. The shape of pans have been listed in model 1.Then based on model 2, the fitness willmigerl of each shape can be obtained. So it can be seen clearly that the shape which have the bigger fitness willmigerl is more suitable for the oven. What's more in order to demonstrate the different outcome of every shape. The relation between p and Z is found for every shape of pans. And this relation is displayed in figures. At last, we make a summary of the relation between p and Z. This relation is also displayed in a figure.

Secondly, only the variation of W/L is considered, the change of p is ignored. As the above, different shapes of pans were placed in the oven. And the outcome is displayed in histogram.

2.5 Result and Analysis to the problem

The size of pans is in [5], and size of oven inside is in [8]. Based on those data, as a result one obtains:

1.Only on the condition one ,that is p=1,it means that the aim becomes to realize the maximum area utilization rate. Owing to the oven is rectangle and the pan shape can change, so the rectangle pan must reach the aim according to oven's W/L. The result is as follows:

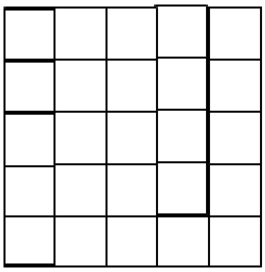


Figure.4 The configuration of rectangular pans in the oven

2.Only on the condition two, that is p=0,it means that the aim becomes to realize the maximum even extent u. According to the conclusion from the model 1,it's easy sure that round pan can reach the aim incontrovertibly. The result is as follows:

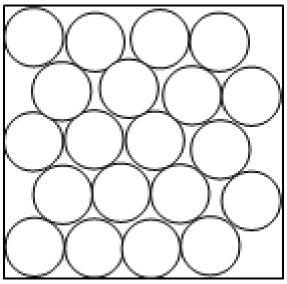


Figure.5 The configuration of round pans in the oven

3.On condition (3), the even extent of heat distribution and area ratio are both taken into account:

(1) results when only vary W/L:

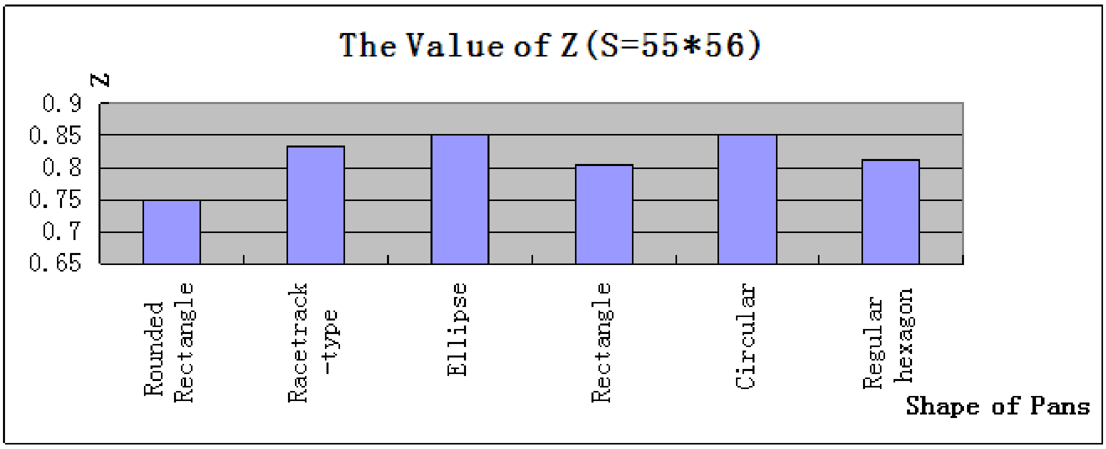


Figure.6 The Value of Z(W/L=55/56)

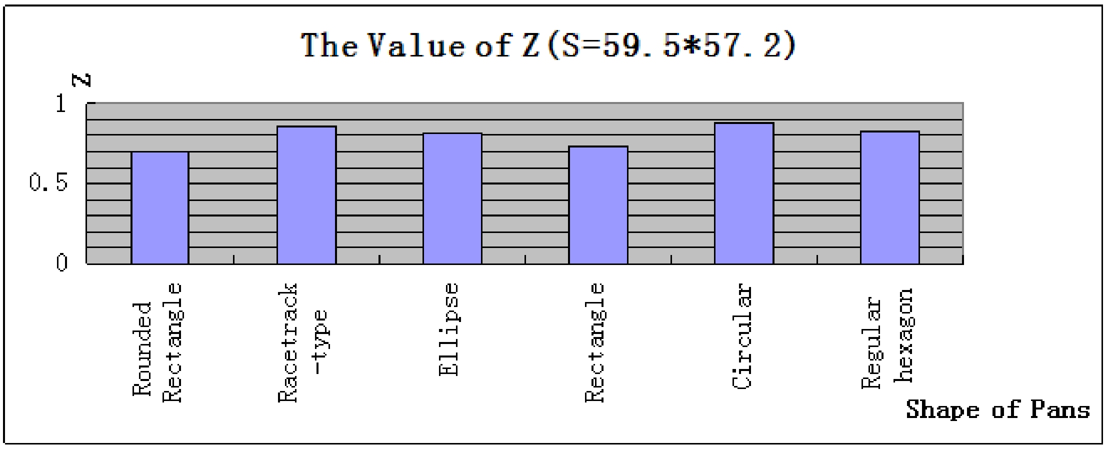


Figure.7 The Value of Z(W/L=59.5/57.2[9])

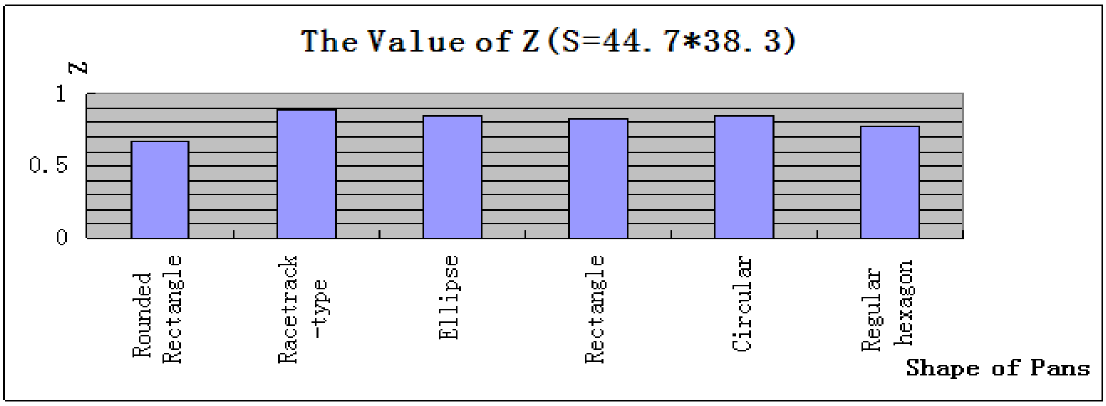


Figure.8 The Value of Z(W/L=44.7/38.3[10])

The three figures above shows the Z value of different pan shape when we vary W/L only. On this occasion, the weight p is set at 0.5.

Fig.6 is the result when the size of oven is 55*56. From Fig.6, the Z value of circular pan is the biggest , it means that circular pan is the best selection under this circumstance.

Fig.7 is the result when the size of oven is59.5*57.2. From Fig.7, the Z value of circular pan is the biggest , it also means that circular pan is the best selection on this occasion.

Fig.8 is the result when the size of oven is 44.7*38.3. From Fig.8, the Z value of Racetrack-type pan is the biggest , it also means that Racetrack-type pan is the best selection on this occasion.

(2) results when only vary the weight p

The following tables show the configuration of pans in the oven based on the result of model 2:

Table.6 The configuration of rectangular pans

	Rectangle			
size（b*a）	10*10.8	9*12	8*13.5	6*18
variance	0.2732	0.2624	0.2828	0.3014
number	25	26	25	27

Table.7 The configuration of Racetrack-type pans

	Racetrack-type			
size（b*a）	6*9.93	7*7.22	8*4.935	9*2.95
variance	0.2043	0.2224	0.1991	0.1922
number	27	24	24	24

Table.8 The configuration of rounded rectangle pans

	Rounded Rectangle					
size（a*R）	5.5*1	6*1	6.5*1	7*1	7.5*1	8*1
variance	0.2637	0.2461	0.2394	0.2312	0.2378	0.2562
number	22	21	21	18	20	16

Table.9 The configuration of ellipse pans

	Ellipse					
size(a*b)	3*11.5	3.5*9.8	4*8.6	4.5*7.64	5*6.88	5.5*6.25
variance	0.3912	0.3022	0.2446	0.2131	0.183	0.1829
number	18	20	18	23	20	20

Table.10 The configuration of circular and regular hexagon pans

Circular	
size(R)	5.9
variance	0.1829
number	20

Regular hexagon	
size(a)	6.44
variance	0.1979
number	20

There are some configuration figure of pans in the oven, as follows:

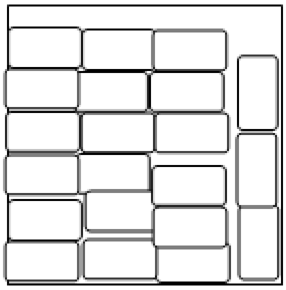


Figure.9 The configuration of rounded rectangular pans in the oven

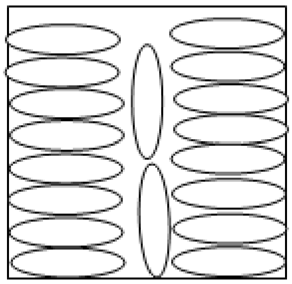


Figure.10 The configuration of ellipse pans in the oven

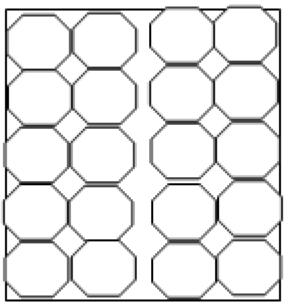


Figure.11 The configuration of regular hexagon pans in the oven

The relationship between Z and p is displayed as the following figures.

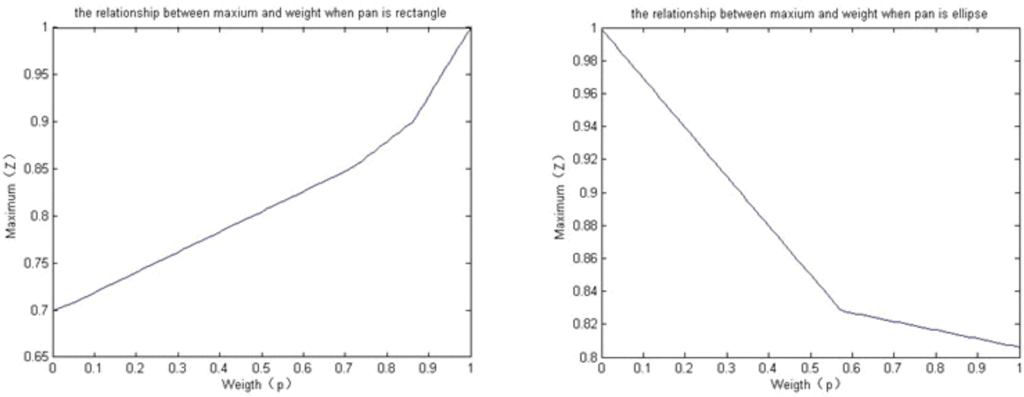


Figure.12 The relationship between Z and p (rectangle,ellipse)

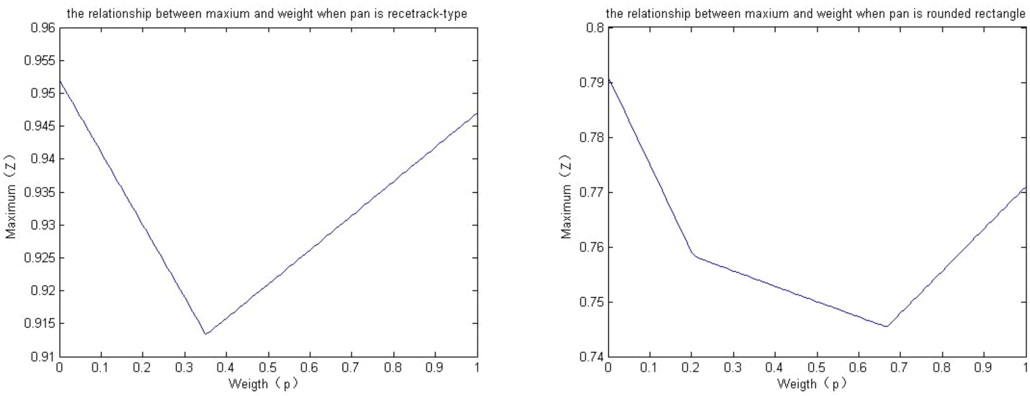


Figure.13 The relationship between Z and p (racetrack-type,rounded rectangle)

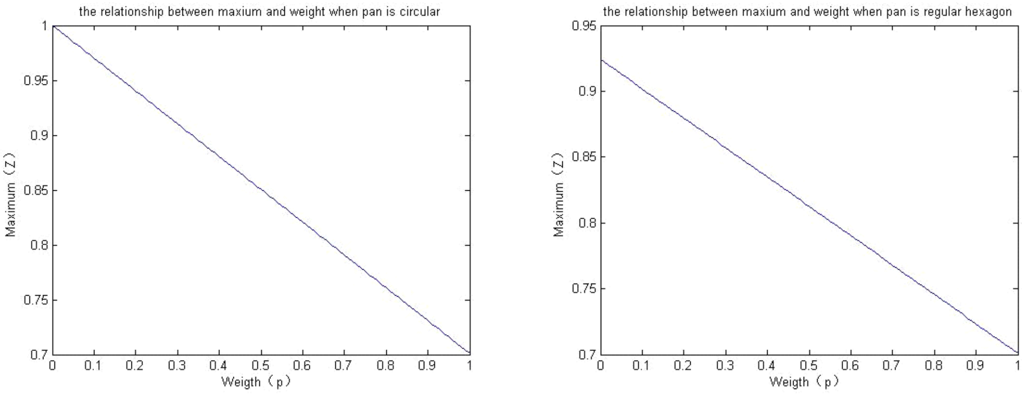


Figure.14 The relationship between Z and p (circular,regular hexagon)

The six figure above shows that the relationship between Z and p. And as a matter of fact, it really makes sense. For instance, in Fig.12, from the relationship between Z and p of rectangle, Z becomes bigger while p increases. When $p=0$, Z is nearly equal to 0.7.On the other hand, when $p=1$, Z is nearly equal to 0. Next, let's come to analyze this phenomenon. In model 2, p stands for the weight of area ratio v. According to the actual situation, if $p=0$, it means that the area ratio is not taken into account. The objective function can be transformed to:

While $p=0$, in Eq.(20), yields: $\max Z = u$ (22)

In this way, it means that the consumer is only concerned about the uniform heat distribution. From the work above , the even extent u of rectangle is nearly to 0.7. So it can be seen that the theoretical results is accord with the actual situation. According to the actual situation, if $p=1$, it means that the even extent is not taken into account. The objective function can be transformed to :

While $p=1$, in Eq.(20), yields: $\max Z = v$ (23)

Under this circumstance, it means that the consumer only concerned about the area ratio v. From the work above, the area ratio v of rectangle can reach nearly 1 if the right size of rectangle pan is selected. So the theoretical results in Fig.15 is accord with the actual situation.

Follow the analysis step above, it can be proved that the relationship between Z and p of circular pan is correct. And the trend of other figure make sense.

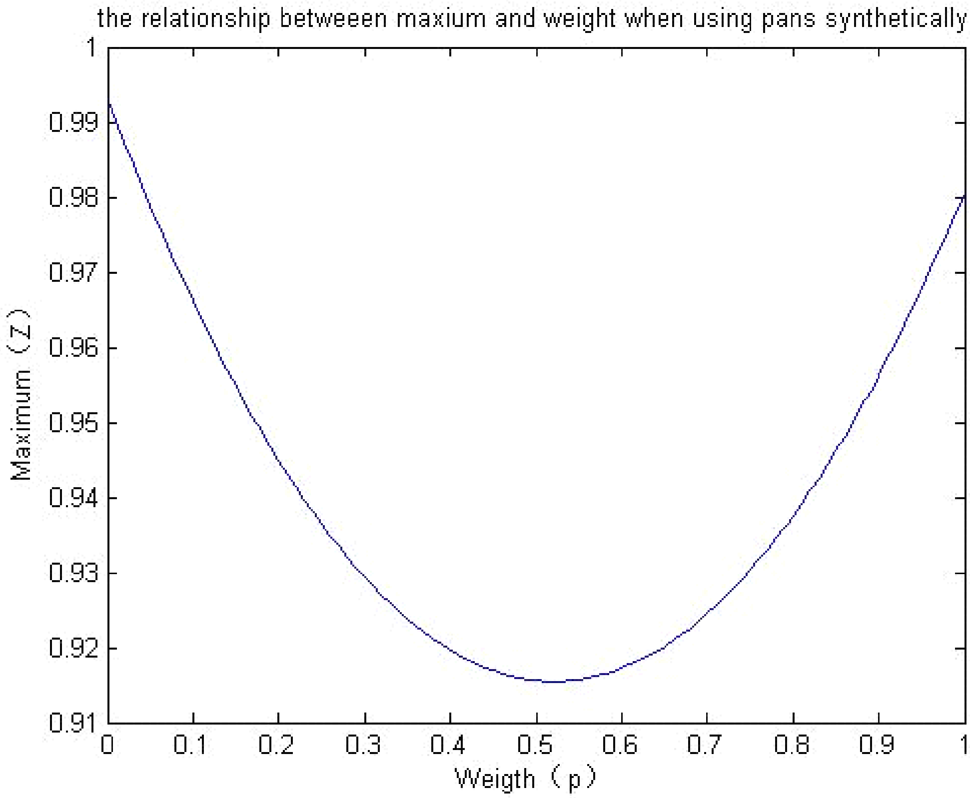


Figure.15 The relationship between Z and p

2.6 Strength and Weakness

Model 2 is a mixed model of two-dimensional packing model and combination optimization model. In this model, the packing step and optimization step were combined into one model. So it can reflect the actual situation well. And in this model, different weight p can be assigned to the area ratio v and even extent, so it can select the best pan shape for oven under different situation.

But on the other hand, this model also has several weaknesses. The most important one is that this model is a non-linear model, the solution for this model is a bit complex. We have to simply the solve procedure. It have been discussed in the solution for model 2. Therefore, the outcome of this model can't be globally optimal solution.

2.7 Conclusion

Model 2 is a mixed model of two–dimensional packing model and combination optimization model. In this model , we vary the weight p and the W/L of oven to select the best shape of pan for the oven. In order to make the model more convincing, we use several different shape of pans in the model.

Next, we obtain the relation figure of p and Z and the Z value of different pan shape when we vary the W/L.

Finally, we compared the theoretical results with the actual situation and use them to select the best pan shape under different circumstance.

III Future Work

3.1 The improvement of model 1

Model 1 is based on heat conduction equation and heat convection. But actually, when baking product, thermal radiation also exists. So thermal radiation should be added into the model to make it more precise.

3.2 The improvement of model 2

Model 2 is a mixed model of two-dimensional packing model and combination optimization model. But the two-dimensional packing in this paper is intended for rectangle. So it's necessary to find algorithm that can be applied to more shapes.

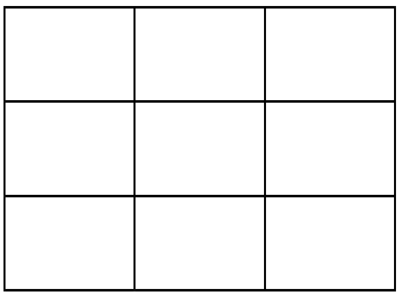
IV A letter

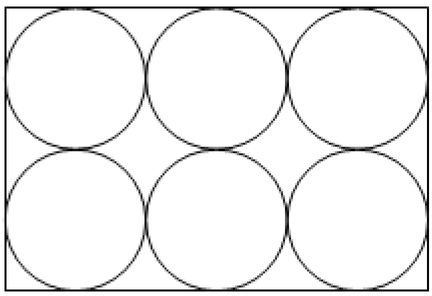
Dear Editor:

We take part in the 2013 MCM during the last four days. The problem we study during these days is the optimization of Brownie Pan. As it known to all, when baking in a rectangular pan, the product gets overcooked at the corners. So it is necessary to change the shape of pans to get better product quality.

However, there is a contradiction between product quality and the space use ratio of the oven. We will explain it in a common way. For example, it is common sense that product gets overcooked at the corners of the rectangle. On another occasion, when we bake product in round pans, the product is heated evenly at the edges. So you may think that why not we bake product in round pans. Then the quality and taste of the product will be better.

Unfortunately, every coin has two sides. Although the product is heated evenly at the edge of round pan. When we use round pan instead of rectangle pan, the space use ratio will decrease. It is illustrated in the following figures





.

Fig.1 use rectangle pans in oven Fig.2 use round pans in the oven

Fig.1 and Fig.2 shows that the space use ratio of rectangle pan is bigger than the round pan.

In general, we have three choices when baking product. If you are only concerned about the quality of product, you can choose the round pan. If you are only concerned about the space use ratio of the oven, you can select the rectangle pan. In addition to these, you can make a compromise. For example, the space use ratio of rounded rectangle is bigger than rectangle. While the heat uniform degree of rounded rectangle is smaller than circle.

In particular, if you want to design a pan that is most suitable for yourself. Our mathematical model will provide a good solution for you.

V References

[1] J. Kidawa-Kukla: Temperature distribution in a rectangular plate heated by a moving heat source, International Journal of Heat and Mass Transfer 51 (2008) 865–872.

[2] J. Kidawa-Kukla, Analysis of beam vibration induced by moving heat source, in: H.A. Mang, F.G. Rammerstorfer, J. Eberhardsteiner, J. (Eds.), Proceedings of the Fifth World Congress on Computational Mechanics(WCCM V),July 7-12,2002,Vienna, Austria. Vienna University of Technology, Austria, ISBN 3-9501554-0-6, <http://wccm.tuwien.ac.at>.

[3] J.V. Beck, K.D. Cole, A. Haji-Sheikh, B. Litkouhi: Heat Conduction Using Green's Functions, Hemisphere, New York, 1992.

[4] Song E, Li Shiguo: Technique and application of thermal field analysis for electro-thermal products based on ANSYS,Machinery Design and Manufacture, (2005), Number 10, 117-119.

[5]http://baike.baidu.com/view/977867.htm (Feb.02nd-04th,2013).

[6] Aleksandar SAVIC, Tijana SUKILOVIC, Vladimir FILIPOVIC:SOLVING THE TWO-DIMENSIONAL PACKING PROBLEMWITH m-M CALCULUS, Yugoslav Journal of Operations Research:21 (2011), Number 1, 93-102.

[7] Wang Yanfang: An automatic packing algorithm aimed to solve the packing problem in the crystal solar cell fragment encapsulation, Nankai University, (2009), 27-28.

[8]http://detail.tmall.com/item.htm?spm=a230r.1.10.18.WGNJ7G&id=12426809377&adid=&am_id=&cm_id=&pm_id=(Feb.03rd,2013).

[9]http://item.taobao.com/item.htm?spm=a230r.1.10.36.n3Eyt1&id=15640151069(Feb.04th,2013)

[10]http://item.taobao.com/item.htm?spm=a230r.1.10.77.BmXuIX&id=15535918101 (Feb.04th,2013).