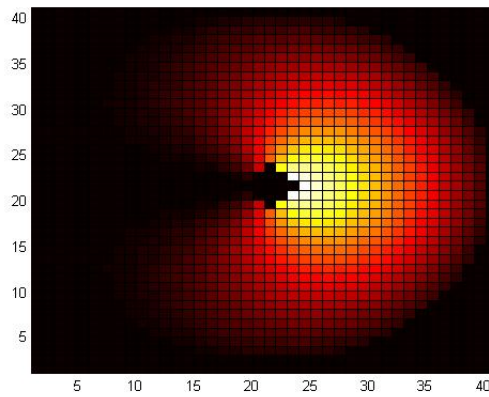


In our paper, we seek to remedy the problem of ineffective search for lost aircraft. We detail a model that utilizes a powerful statistical algorithm known as the Bayesian Search Method, and offer several improvements on the classical version of this procedure. We quantify these improvements by giving data about how they reduce the time it takes to locate a downed plane. Finally, we run our model given data from the recent Indonesia AirAsia Flight 8501 event where a plane was lost, and demonstrate that we find the plane in the same amount of time that it took the real search team.

We conclude that our model is performant, and can be used in real-world scenarios to recover the wreckage of lost planes in a timely manner. Furthermore, our model implements improvements to the classic Bayesian Search Method, which allows searches to succeed faster in practice. Since the classic Bayesian Search Method is currently used in real search systems, we believe these improvements make our model superior.



An initial probability distribution for the location of a downed plane used by our model.

# Application and Advancement in Bayesian Search Theory on Finding Lost Planes

Control #38721

February 9, 2015

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## 1 Introduction

On March 8, 2014, Malaysia Airlines Flight 370 departed from Malaysia on course for Beijing. It would never make it to its destination, and less than two hours after takeoff, the radar made the last known detection of the aircraft, indicating that it was over 200 miles northwest of Malaysia – far off its charted course. The ensuing search for the aircraft became a massive multinational effort, the largest and most expensive in history. At the time of this writing, the location of Malaysia Airlines Flight 370 has remained elusive, and the search is still ongoing.

Airplane crashes are rare, and cases like Malaysia Airlines where the wreckage is never found are rarer. Despite this, we ought to be able to find the location of a crashed plane, if only so that we can learn how to avoid in the future the circumstances in which the crash occurred.

In this paper, we provide a model that tells rescue operations a procedure for how to search for downed planes. We use Bayesian Search Theory to inform our model, which uses initial hypotheses about the downed plane, and statistical methods to provide an accurate and continuously updating guess for where to search next. We give an explanation of the details of the Bayesian Search Method. This is followed by a discussion on how to choose initial hypotheses, and the resulting model. Then we provide several extensions to this basic model, which make the search even more accurate. Finally, we will apply our model to the Indonesia AirAsia Flight 8501 passenger flight which crashed in December 2014, and was recovered days later. We apply our model to this and successfully find the flight in the same amount of time.

## 2 Statement of Problem

Our effort here is to aid the search and rescue operation by providing a concrete method of searching the crash area. To this end, we must make several simplifying assumptions about the conditions of our search operation. This model focuses on plane crashes in the middle of large ocean bodies. We also focus on searching the surface of the sea for floating wreckage with search planes, and ignore the long-term search effort for planes underwater. Additionally, we assume there is no signal or contact from the downed plane. These factors, when combined with an imperfect search plane, lead to the use of an adaptive search method based on Bayesian statistics.

### 2.1 Assumptions

#### 2.1.1 Location of Crash

We assume the crash search area is located complete within the boundary of a large ocean. Most of the surface of the Earth which is covered by land is populated to some degree, finding a plane that crashed over land is often much simpler than finding one lost at sea. The oceans cover a large percentage of the

surface area of the Earth and have the added challenge of being a constantly moving surface with large swells and other effects.

### 2.1.2 Ocean Currents

According to charts released by NOAA [6], the ocean currents are for the most part very slow, about 15 cm per second. Our model is dealing with distances orders of magnitude higher, when we separate the search area into grids it would take over 8 days of searching for the current to move something from one grid into the next, our searches were terminated anywhere from a couple of days to several weeks, but even in the worst case the elements in a 40 by 40 grid would have only shifted 3 squares, a negligible amount overall.

### 2.1.3 Crash Site vs. The Plane Itself

We assume that it is good enough to find the floating debris from the crash site. Once these are found the amount of information available greatly increases and search efforts can be localized. We solve the problem of using search planes to find any sign of the initial wreck over a vast area rather than the easier problem of finding the wreckage once the crash site is known. In most cases where debris is found, the plane itself is found not long after, while when nothing is found, searches last orders of magnitude longer.

### 2.1.4 No Additional Contact

We assume that the only contact we had with the plane was when prior to it being lost, we have no black box signal and no additional way to remotely gain information from the downed plane. In cases where the black box is giving off a ping it is much easier to locate the wreckage, we want to solve the problem of a complete disappearance. We want the best chance to find a downed plane with as little information as is reasonable.

## 3 Bayesian Search Method

At the heart of our model lies Bayesian Search Theory, an application of statistics that is used to search for lost items. This method has been used to successfully recover lost vehicles since its introduction in the mid 20th century. Notable examples of this include finding the USS *Scorpion*, a nuclear submarine lost by the United States navy in 1968; locating the wreckage of Air France Flight 447, an airplane that went down in 2009; and even discovering the resting place of the S.S. *Central America*, an American passenger ship that sank in 1857 [5].

### 3.1 Optimal Methods of Search

There are many ways to search for lost objects. Suppose that a child loses his ball in a field with tall grass. How should he structure a search to find it? One

way someone might suggest is to start at a corner of the field, walk all the way up one side, and when the opposite edge is reached, move slightly inward, and walk back down to the bottom of the field. Repeating this procedure until the ball is found or the entire field has been walked. If the child is diligent enough and the ball is truly in the field, then this method cannot fail. The downside of this method is that it is very slow, sometimes requiring that all or almost all of the field be searched before we find the ball. So we might wonder if we can do better.

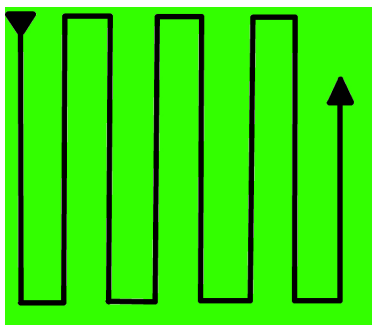


Figure 1: Simple method of searching an area

Now consider the same problem, but imagine that the field being searched has many hills. We know balls will roll down hills, so we conclude that the location of the child's lost ball is much more likely to be at the lower-elevation places in the field. So if lower-elevation areas of the field are searched first, it is likely that the ball will be found more quickly than in the first method.

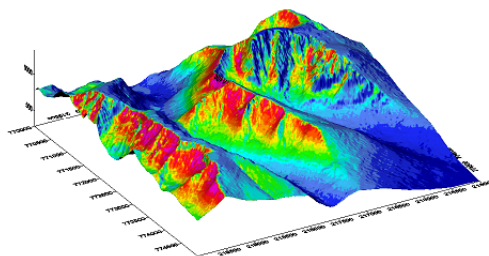


Figure 2: Heatmap showing elevations

This is the idea behind Bayesian Search Theory. Prioritize searching in places where the lost object is likely to be, and use the information gained from previous failed searches to update where these most likely places are.

### 3.2 Overview of the Algorithm

The general description of the algorithm is as follows.

1. Think of reasonable guesses for what happened to the lost object, and construct a probability distribution for each one. These will be functions that, given a position, gives an indication of how likely we are to find the object at that position.
2. Create a function that takes in a position, and returns the probability that we find the object if we look at that position, given that the object is actually there. For example, things are easier to find in clear weather as opposed to stormy weather, so if the lost object is currently in a location undergoing a storm, we may still fail to find it with high probability even though it really is there.
3. Combine these probability distributions to get an overall map of the area, where we can identify which areas have the highest probability of our object being found if we search there. Then begin searching the highest probability areas first.
4. Revise these probabilities after every search using the well known Bayes' Theorem. Abstractly, if we search in a location and don't find our object, then the probability that our object is around that location is reduced, and the probability that it is in other locations is increased.

### 3.3 Bayes' Formulas

Suppose a location has a probability  $p$  of containing our lost object, and that the probability of successfully detecting the lost object if it is in that location is  $q$ . If we search the square and do not find the lost object, then using Bayes' Theorem we can update  $p$  to  $p'$  where

$$p' = \frac{p(1-q)}{(1-p) + p(1-q)} = p \frac{1-q}{1-pq} < p.$$

And for each other location, if its probability before the search was  $r$ , then its probability after the search is given by  $r'$  where

$$r' = r \frac{1}{1-pq} > r.$$

## 4 Initial Distribution

Before we begin applying the Bayes' Search Method, we must first come up with reasonable hypotheses about what could have happened to the downed plane. For each one of these hypotheses, we can form a probability density function, that given a location, returns an indication of the probability that the

lost plane is at that location. Once we have all of these, we can combine them to form our final density function. We will use this final combination as our initial distribution when we begin to apply the Bayes' Search Method.

## 4.1 Forming Hypotheses About Lost Planes

Given our assumptions, the downed airplane we are searching for cannot transmit signals. Because of this, we are only able to rely on the last contact an external entity had with the aircraft. This is typically a radar detection. In the case of Malaysia Airlines Flight 370, the last contact was when the plane left the range of Malaysian Military radar while over the Andaman Sea [2]. This last radar detection gives us important information about the lost plane, such as its position, altitude, speed, and direction at last contact. From this information we can form two generally reasonable guesses for how the plane behaved in the time immediately following. We detail these guesses in the next sections.

### 4.1.1 Hypothesis 1: Optimistic Distribution

The first hypothesis we consider is what we will refer to as the “optimistic” distribution. The intuition for this distribution is that after the point when the plane is no longer detected by radio and we lose it, it continues to fly until it runs out of fuel, at which point it crashes into the ocean. So the plane flies as far as possible. Using the plane's altitude, speed, direction, and flight duration information that we get from the final radar detection, as well as knowledge about the model of the plane, we can compute how long and far the plane can fly from that point before running out of fuel. Call this maximum distance  $M$ . Then the optimistic distribution will assign the highest probabilities to the location a distance  $M$  away in the direction that the plane was travelling as determined from data gotten from our last radar detection.

Because it is enlightening and will aid explanation, we provide the explicit probability distribution formula along with explanation. Let  $\rho_o$  be the unnormalized formula for the optimistic distribution. Then

$$\rho_o(r, \theta) = r \cos^2(\theta/2), \quad r \in [0, M], \quad \theta \in [-\pi/2, \pi/2].$$

So when  $r$  is 0, we are at the point where we lost contact with the plane, and  $\rho_o(0, \theta) = 0$ . When we are at  $\rho_o(M, 0)$ , we are at the point distance  $M$  away from where we lost contact, having traveled at the same angle as the plane was when it lost contact, and  $\rho_o$  gives us the largest probability here. As the angle  $\theta$  deviates from the angle at which the plane was travelling,  $\rho_o$  decreases the probability that the plane has landed at these locations, because our hypothesis is that it is more unlikely that the plane drastically changed the direction it was flying.

Another property that we have gleaned from this distribution is an upper bound for how far the missing airplane can have traveled. Notice that  $\rho_o$  assigns no probability to the region outside the circle of radius  $M$ . That is because the



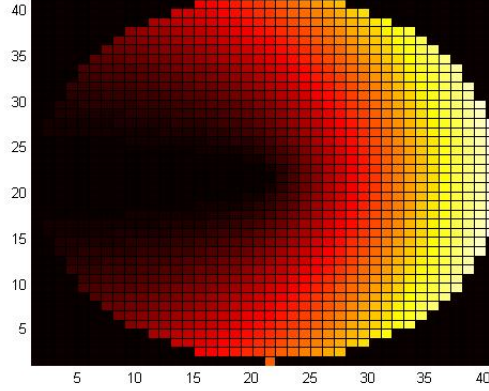


Figure 3: Optimistic distribution. Bright colors are high probability, dark are low.

plane absolutely would have run out of fuel before reaching that far, and so we have obtained a bounding circle within which to contain our search.

#### 4.1.2 Hypothesis 2: Pessimistic Distribution

The next hypothesis we detail is what we have dubbed the “pessimistic” distribution. The idea for this scenario is that the plane crashed into the ocean very soon after our last radar ping. Eastern Airlines Flight 401 encountered this scenario in 1972 when their autopilot became accidentally switched off, and because it was too dark to see, and the plane descended so gradually, nobody noticed the problem until the plane hit the ground [4].

The pessimistic distribution is constructed by looking at the direction that the plane is traveling at the time of the last radar contact, as well as its speed, and altitude, and imagining that the plane is turned off immediately after the last radar contact. So the plane glides for a little while until it hits the ocean. This is effectively what happened to Eastern Airlines Flight 401.

Let  $m$  be the distance the plane travels once it stops working immediately after the last radar detection. So after this last radar ping, the plane glides a distance of  $m$ , before hitting the water. As before, we provide the explicit formula for the un-normalized pessimistic probability distribution,  $\rho_p$ . Let

$$\rho_p = (M - r) \cos^2(\theta/2), \quad r \in [m, M], \quad \theta \in [-\pi/2, \pi/2].$$

When  $r = m$  we see that the probability is highest of finding the plane with this distribution. When  $r = M$  the probability is lowest, since  $\rho_p(M, \theta) = 0$ . And as the angle  $\theta$  deviates from the original course of the plane,  $\theta = 0$ , the probability becomes worse and worse that the plane has landed there, since we consider it unlikely that lost planes change course drastically.

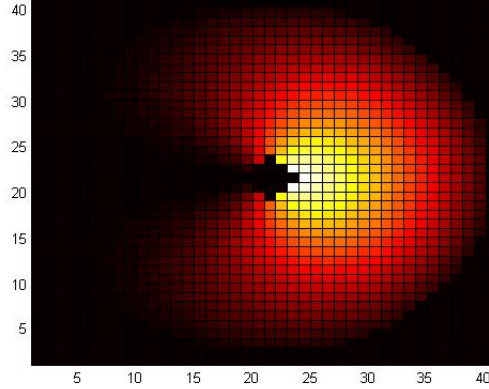


Figure 4: Pessimistic distribution. Bright colors are high probability, dark are low.

## 4.2 Combining for a Final Distribution

We have constructed two distributions, our optimistic distribution  $\rho_o$ , and our pessimistic distribution  $\rho_p$ . In order to use these with our model, we need to combine them in some coherent way. We obtain a final distribution by multiplying both of these distributions with some weight scalar, and summing the resulting products. So our final distribution is given by  $D$  where

$$D = w_o \rho_o + w_p \rho_p$$

for some weights  $0 \leq w_o, w_p \leq 1$ , such that  $w_o + w_p = 1$ . So the only question that remains is how to select what the values of these weights should be.

### 4.2.1 Selecting Appropriate Weights

If we do not have any extra information we propose the following scheme for choosing weights. At the last radar detection of the lost plane, we learn its altitude. Call this value  $A$ . We also know the model of the plane, and so we can look up the typical cruising altitude of these types of planes. Call this  $A'$ .

The intuition for choosing weights is that if  $A$  is close to zero, then the plane is close to the ocean, and we expect that it will crash soon. Alternatively, if  $A$  is close to  $A'$ , then we expect that it will continue for much longer, possibly until it runs out of fuel. So recall that  $w_o$  is the weight for the optimistic distribution  $\rho_o$ . Then let

$$w_o = \frac{A}{A'}$$

and thus  $w_p = 1 - w_o$ . The reasoning is that as  $A$  is close to  $A'$ , the plane was last seen as being very close to normal cruising altitude, so we would expect it

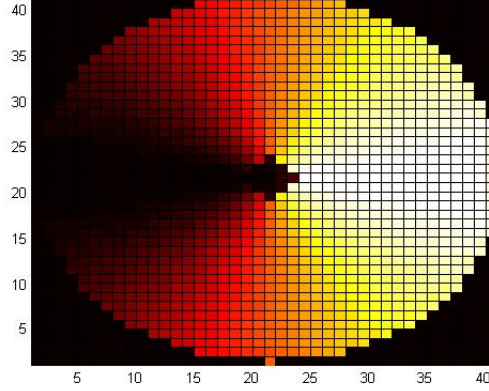


Figure 5:  $D$  with  $w_o = w_p = 0.5$ .

to go farther, so we would want  $w_o$  to be close to 1 and  $w_p$  close to 0, because the optimistic distribution gives a better distribution for this scenario. If  $A$  is close to 0, then  $w_p$  becomes the higher weight, and the pessimistic distribution becomes favored.

## 5 Basic Model

Our model divides the search into a grid of squares spanning the area of ocean where the probability distribution of the crash is nonzero. We split the search behavior into two different length scales: the search effort inside any one square of ocean, and the search effort over all of the squares. This is reflective of how current search and rescue missions operate, with different vessels being sent to "look into" different areas on a map and to report back.

### 5.1 Inside a Search Square

The discretization of the search area in our model is done at a high cell-count and the probability distribution of the position of the plane is broad. This means that the probability of the plane being in any one of our squares is roughly constant across the square, which will be important later.

Each search vessel has an associated probability of detecting the downed plane. This probability should reflect how the detection is done and depend on the distance of the downed plane from our searcher at any given time. One commonly used model for this is an inverse-cube law which characterizes the ability of human sight to see objects in the distance. It is possible to consider even more elaborate detection probability functions incorporating sonar, radar, and other equipment as well. However, the effort in creating such a sophisticated model is

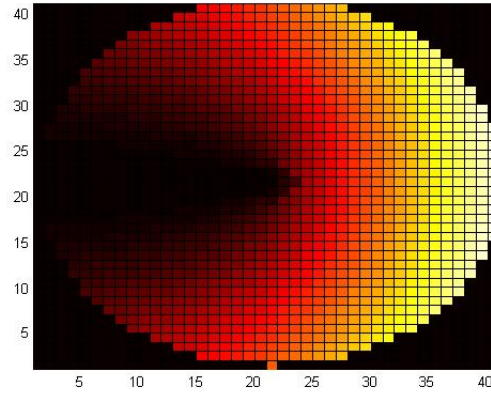
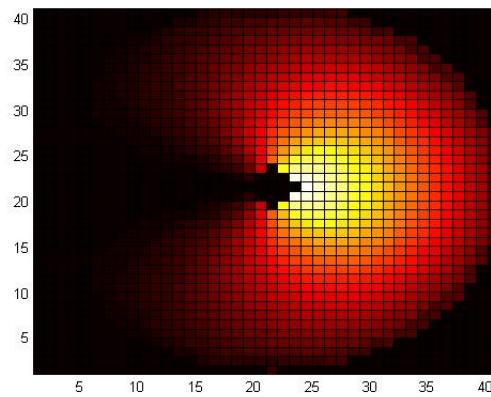
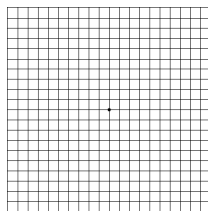
Figure 6:  $D$  with  $w_o = 0.9$ ,  $w_p = 0.1$ .Figure 7:  $D$  with  $w_o = 0.1$ ,  $w_p = 0.9$ .

Figure 8: We subdivide the search area into a grid of squares

washed out by the physical random motions of the plane inside a given cell. In addition, the relative motion between search planes and a downed plane inside a

given unit cell makes an analytic approach intractable. Instead, we consider the lower-bound worst-case scenario of randomly searching inside a given cell for the downed plane.

Let  $p(x)$  be the probability of detecting a lost plane a distance  $x$  away from our search vessel. Let  $W$  be the search width, defined to be  $\int_{-\infty}^{\infty} p(x)dx$ . Physically, our goal is to shift from the detailed description of  $p(x)$  to a much simpler “box” of certain detection with width  $W$ .

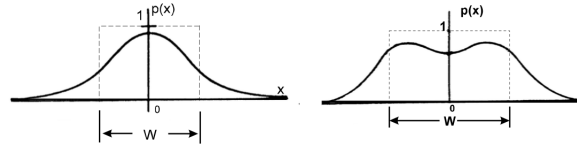


Figure 9: Simplifying the probability of detection

Suppose our search plane travels along a path of length  $L$  divided into  $n$  equal segments of length  $L/n$ . Assuming the probability of the downed plane is evenly distributed in any of the pieces, then our probability of detection distribution  $p(x)$  will sweep an area of detection with the path of the plane. We can convert this hard problem into a simpler one by noting that the total area under the detection curve  $p(x)$  is equal to a box of certain detection (probability of detection 1) of width  $W$ . So the probability of detecting the downed plane for a complex detection distribution  $p(x)$  is the area swept out by our box of certain probability:  $W * (L/n)$ [8]. The probability of detecting the downed plane is then the same in the case of a uniform distribution across the square, which we have. So, probability of the downed plane being in this area is the fraction of the area swept over the total area:  $WL/nA$ . The probability of an observer *not* detecting the downed plane is  $1 - WL/nA$ . This is for each individual step, of  $n$  total steps in the path of length  $L$ . So, the probability of *not* detecting the downed plane in any of the  $n$  patches of covered area is

$$(1 - WL/nA)^n.$$

Thus, the probability of detecting the downed plane after sweeping over the  $n$  patches of covered area is

$$1 - (1 - WL/nA)^n = 1 - e^{-WL/A} = q$$

for large  $n$ . So, all of the complexity of a plane's search capabilities are wrapped up in the size of  $W$ . To assist real searchers, data should be collected on the actual probability of detection for the combination of devices included in each type of search vessel. For demonstration purposes, we have three different types of search planes. The most basic plane has  $W = 20$  miles, corresponding to a searcher using sight with complete detection of a downed plane within 10 miles of either side of the plane. The two more advanced planes in our model have  $W = 40, 80$ .

## 5.2 Search effort across all of the squares

Now that we have a handle on the search plane behavior inside each discrete search cell, we need a method for instructing search planes *which* squares to search. This is done by using the Bayesian search method described in section three. The first model considers the probability distribution of the position of our downed plane and sends a search plane to the square with highest probability. Upon searching that square, the entire distribution lattice is updated with new probability values. The plane is then sent to the new highest-probability square to search. This process is repeated until the lost plane is found.



Figure 10: simple model scheme

## 6 Extending the Basic Model

There are several points on which the model above can be improved.

- There is only a single search plane, and only one type of plane
- The plane teleports between probability maxima

To increase the realism of our search algorithm, we consider three different types of planes with absolute detection  $W = 20, 40, 80$ . We also consider a scheme in which each plane only chooses to move between neighboring cells in our discrete probability distribution, unless the global maxima probability is four times higher than the one being searched. This decision reflects the real world scenario of “knowing” that the downed plane is probably somewhere you are not looking, so the search effort is refocused to a different location.

### 6.1 Multiple Search Planes

When we look at real searches, there are many search planes involved and they can have a wide variety of effectiveness at searching. In order to extend our model we first make a simple change to incorporate a selection of search planes. Instead of the highest probability square being searched each time we take a step

in the algorithm we have a group of search planes which each take a variable number of steps to finish a search of the square they occupy. As time steps, each plane will search for a number of time steps dependent on their absolute detection, then they will update the probability distribution with the results of their search and find the highest probability square which is unoccupied. This will be the next square searched by this plane. In practice this method looks very similar to having just one plane, except the squares are searched faster. The squares are still searched in the same order, as the relative probabilities are still ordered the same and each plane chooses the highest probability square it can. As we can see in Fig. 11, as we add more planes to the search, the time to find the crash site goes down. We also see that the variance decreases with more planes as well, meaning that we are less likely to take an unusually long time to find the site.

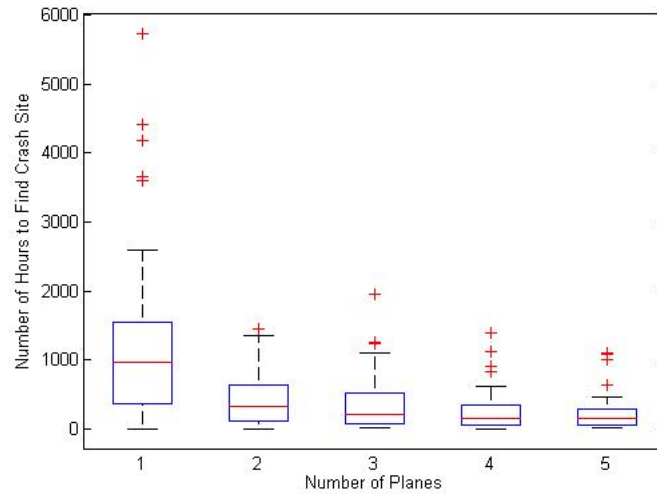


Figure 11: As the number of search planes increases, the spread and time to find the crash site decreases.

## 6.2 Realistic Travel and Search Patterns

Another improvement we can make to our model is to more realistically model plane movement. In the basic model planes search the highest probability square they can, regardless of how far away that square is from their current square. In a realistic search, planes would have to travel from square to square and it would be more efficient to search a local area until the area near you has a low probability. This means that sometimes we will search squares which are not the highest probability squares, but since planes travel in continuous lines this will more accurately represent a real search.

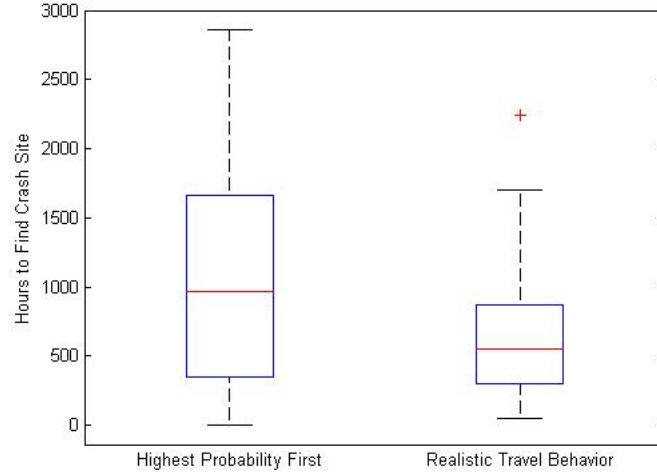


Figure 12: When we apply realistic travel to multiple planes we see that our search performance improves.

When we apply this new search behavior we notice that our performance improves slightly (Fig. 12). In the case of only one plane, the method of searching the highest probability squares first performs better, as we would expect. However, when we use multiple planes a different behavior emerges. Because all of the probabilities are fairly low and distributed over a large area, the planes picked from our distribution made from combining the optimistic and pessimistic distributions turns out to put planes in the medium probability areas quite frequently, as the area under the distribution in the medium areas is larger than the area under the distribution at the maximal points. When we search the highest probability areas first it takes a long time to reach the medium probability areas because you search in order from high to low. When we use more realistic behavior, we end up searching medium probability areas sooner, before moving to higher probability areas. With a distribution that has more peaks and a greater difference between high and low probabilities, this behavior will change. However, in most cases where planes are lost, there is only a small amount of information available, and the distributions we pick as an initial guess look closer to the one we have presented.

## 7 Analysis

As we explored in the last section different behaviors of planes can have significant effects on the mean time to find a crash site. Another big piece of the model is in building the initial guess as to the probabilities of each section of the search



zone. The more accurate we are with our initial guess, the faster we will find the crash site. Figure 13 shows two scenarios, one where we use our combination of optimistic and pessimistic distributions to start our search and the crash site itself is sampled from that distribution. The other, we use our distribution, but the crash site is chosen uniformly at random over the search area.

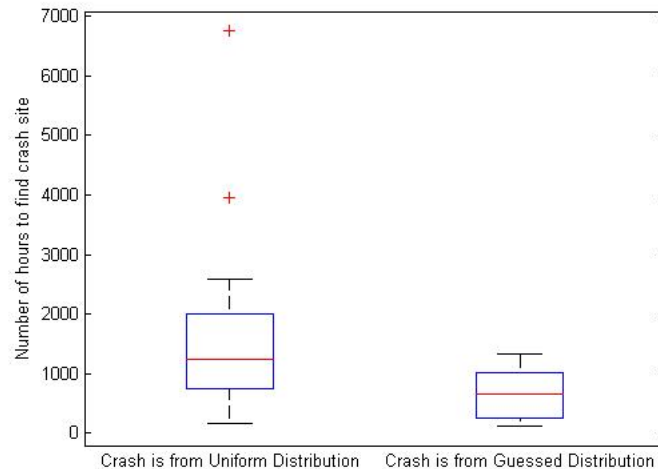


Figure 13: We see a significant difference when our initial distribution matches the actual distribution of the crash site.

This indicates that one of the best ways to improve search efforts is to be as accurate as possible when coming up with the initial distribution. If we put too much weight in the wrong area, the searcher can end up searching a large area with very little actual chance of locating the crash site. Many factors can be used to great effect in creating the distribution. In real world cases information from last contact and last radar pings will be used to determine speed, direction, altitude, possible problems, and many more factors. With this searchers can create reasonable constraints on the search area and by identifying probable causes for the cause of loss of contact they can create probability distributions for each. The distributions we discussed earlier are just two simple cases. These distributions can be very specific to possible catastrophes and then weighted according to the likely hood of the catastrophe. For example, the loss of an engine or wing would have a very specific behavior, but is not very likely. So we can model the behavior and combine it with the initial distribution, but weight it much lower since it is not very likely that such a disaster occurred. The more situations we can quantify, the better our distribution will match the actual distribution for a crashed plane.

## 8 AirAsia Flight 8501

The most recent notable example of a plane becoming lost is AirAsia Flight 8501. The international passenger flight was traveling from Indonesia to Singapore on December 28, 2014, when it crashed in bad weather [3]. Two days after the crash, debris from the aircraft were found in the Java Sea.



Figure 14: Flight path of Indonesia AirAsia Flight 8501 [1].

In this section, we apply our extended model to the AirAsia Flight 8501 situation in order to verify its effectiveness.

### 8.1 Initial Hypotheses About the Plane's Location

Before disappearing from radar, the AirAsia plane descended 1000 feet in six seconds, and turned sharply to the left, rotating in at least one complete circle [9]. This behavior is consistent with the plane stalling, which is what happens when a plane tries to gain altitude too quickly and its engines turn off. This kind of information is critical for forming useful hypotheses for our model.

Given that the plane seems to be rapidly descending at last contact, we might suspect that the pessimistic distribution is appropriate for modeling this behavior. However, the pessimistic distribution relies on knowing that the plane is going in a specific single direction, and that is not the case here. The AirAsia plane rotated very quickly during our last radar detection, so it's not clear in which direction to accurately point the pessimistic distribution. By definition, the pessimistic distribution will point in the last known direction of the plane, but it appears that this is not a very trustworthy way to model the situation.

Our approach is to add another distribution to model the situation. From last radar contact with the flight we knew that the plane was losing altitude quickly and had changed heading by over 360 degrees, the start of a spin. In order to obtain a better distribution suited to the information available in this case we look at the minimum speed required to not stall in the Airbus A320, this speed is often known as  $v_{\min}$ . From this we estimated the airspeed of the plane to be around 170 knots. This would give the plane about a 7 mile radius area to fall into from its cruising altitude. However, it is likely the pilot tried to fly out of the stall by point the nose down to gain speed and then pulling up to regain control, to account for this we widened the radius by about another 7 miles to create a gaussian distribution centered around the point of lost contact with a standard deviation of about 14 miles. A gaussian was the best choice because we had little idea which direction the plane was facing, last information told us it was probably in a spin, so any direction is equally likely.

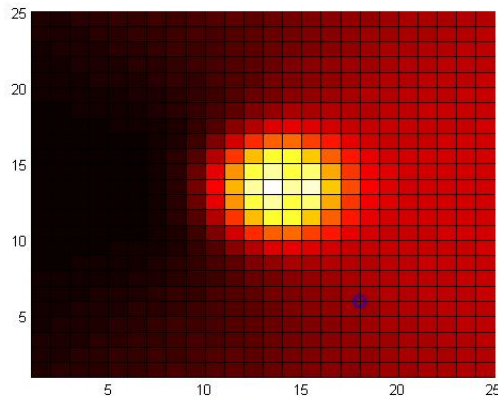


Figure 15: Initial distribution for AirAsia search. The blue circle is the downed plane.

## 8.2 Running the Model

We have our initial distribution, so we can run our simulation. The AirAsia flight was found about 100 miles from the point where radar contact was lost. Our optimistic and pessimistic distribution take into account the plane's heading at time of lost contact, so in order to account for the unknown direction of the plane we chose a random point approximately 100 miles from the point where contact was lost. The following figures show the distribution at 60 steps into the simulation, which is six hours, then at 120 steps which is 12 hours, then at the finishing state of 327 steps, which is 32 hours.

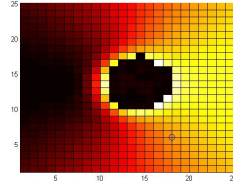


Figure 16: Distribution at 60 steps, 6 hours.

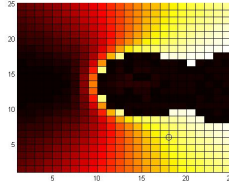


Figure 17: Distribution at 120 steps, 12 hours.

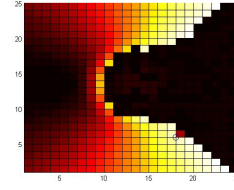


Figure 18: Distribution at 327 steps, 32 hours.

### 8.3 Results

The simulation ran for 327 steps, where each step is 6 minutes. So in all, we searched for 32 hours until we found debris from the AirAsia flight. If we assume that search parties take breaks at night because it's dark, then one eight hour break per day means that the search lasts for 2 days. This is the same amount of time that it took the actual AirAsia search to find debris from the plane. So our model has performed just as well as the real search method used.

## 9 Conclusion

Our approach to modeling this problem was to break it into two major parts: the initial distribution and how information is shared during the search process. We apply our model to the real world example of AirAsia Flight 8150 and also compare the effectiveness of different types of search patterns within the model. In any modeling problem, a solution is limited by the available information. It is very important to assess everything that is known when working with as little information as possible.

We found that one of the most important features of the model was creating an accurate initial distribution for the search area. Searches conclude much quicker and more consistently when the initial distribution matches the actual distribution of the crash. Because we get no information after the plane goes down, we must use our knowledge of airplane mechanics and previous crashes to construct a probability distribution of where the plane is likely to be. Using this distribution, we can constantly reevaluate our probabilities as we search different areas. Each area searched provides us with more information to help us shape the most effective search area. We constantly update because we start with only a minimal amount of information, and even knowing how likely we are to successfully find a plane is useful when combined with results from the field. These can make a large impact on how accurate our distribution becomes over time.

This feature is also one of the weaknesses of the model. If the initial distribution is not close to the actual distribution, then time and other resources will be wasted searching in non-optimal parts of the search area. Our model is at its

best when we effectively use our previous knowledge to inform our search. The biggest strength of using Bayes' method is that information is sparse and we use it effectively. The key to a successful search is being efficient. As time passes, it only becomes more difficult to find a downed plane's wreckage, since currents begin to have a significant effect and the search party's resources dwindle.

Our model was able to closely recreate the scenario of AirAsia Flight 8150 and find the missing plane in a comparable time to the real search. By tweaking the parameters and formulating a good initial distribution we could even improve on that time. This demonstrates its potential and our analysis shows that it even performs better than expected when compared to Bayes' Method over multiple search planes on some likely distributions.

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## For Use in Press Release

There are two major parts to keeping an effective crash search going: making a good guess about where the crash site is and keeping searchers updated to the overall progress.

The initial guess is key to an effective search. If a good guess is made, then search times can be cut dramatically, if a bad guess is made, then a lot of time and resources will be spent looking in the wrong place. A key step airlines and governments can make is to keep detailed records of any type of problem that flights have faced. Quantifying how often mechanical problems, pilot errors, or other anomalies occur can, in the event of a crash or otherwise lost plane, help searchers create a good guess for the initial search area. Even if the problems, errors, or anomalies do not have major effects, knowing how often they occur and how pilots react can provide a rich data source from which to pull from when something bad does happen.

The importance of the initial guess can be illustrated by looking at two recent crashes. AirAsia Flight 8501 was lost in bad weather on December 28, 2014. When the flight lost contact with radar officials knew how high the planes was, how fast it was going, and they even were able to determine that the plane's heading had started to spin, over one full rotation. From this information coordinators were able to determine where the plane was likely to be found, and it was found two days later. On March 8th, 2014 Malaysia Airlines Flight 370 was traveling from Malaysia to Beijing, but only two hours after take off, the plane made last radar contact. Using the available information search efforts began. After the search was underway it was revealed that military radar had made contact after air traffic control had lost contact. This information revealed that the flight had been spotted in a completely different location. The search efforts so far have uncovered nothing due to this lack of knowledge. Keeping detailed data and sharing information about radar contact is crucial to being able to efficiently coordinate a search.

The initial choices on where to spend search effort is made with this starting guess, but information is accumulated about the lost plane's location as a search is conducted. If an area is searched and there is no sign of the crash, then the area just searched is less likely to have the crash site. However, that search area shouldn't be discounted entirely, since search planes are not guaranteed to find the crash even if it is where they are looking. Weather, time of day, and a variety of other factors can make identifying the crash site very difficult. Every time an area is searched, more information about where the downed plane is likely to be and where it is not likely to be is received. This information will feed back into the decisions on where to spend resources, and demonstrably leads to a faster successful search.

Combined, these strategies give us the most crucial improvement we can have for search and rescue operations: decreasing the time to find the lost plane. At the very least, we save more resources of the searching organizations with this method. In the best case scenario, we might even save lives that otherwise would be lost with a different search method.